Crawshaw Academy

Knowledge Organisers

Year 9

A framework for effective
home learning

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 9 HALF TERM 1 (Autumn 1) :

N11 - PROPERTIES OF NUMBER

N12 - PERCENTAGES

G7 - AREA AND VOLUME

YEAR 9 — AUTUMN

N11 - PROPERTIES OF NUMBER

Sparx Maths

Factors, multiples and primes – M823, M322, M227, M698
Write a number as a product of prime factors – M108 Use prime factors (E) – M365
Highest common factor (HCF) and lowest common multiple (LCM) – M365, M227, M698
Venn diagrams – M829 Use a Venn diagram to calculate the HCF and LCM – M365, M829
Integers, real numbers and rational numbers – M763, M527, M704, M522
Introduction to surds (E) – M135, U338, U633

What do I need to be able to do?

Step 1 Factors, multiples and primes
Step 2 Write a number as a product of prime factors
Step 3 Use prime factors (E)
Step 4 (HCF) and (LCM)
Step 5 Venn diagrams
Step 6 Use a Venn diagram to calculate the HCF and LCM
Step 7 Integers, real numbers and rational numbers
Step 8 Introduction to surds (E)

Keywords

Multiples – Numbers obtained by multiplying a number by an integer.
Factors – Numbers that divide exactly into another number.
Prime – A number with only two factors: 1 and itself.
Prime factorisation – Writing a number as a product of prime numbers.
Square – A number multiplied by itself (e.g. $4 = 2^2$).
Cube – A number raised to the power of three (e.g. $8 = 2^3$).
Root – The inverse operation of powers (square root, cube root).
HCF – Highest Common Factor; largest factor shared by two numbers.
LCM – Lowest Common Multiple; smallest multiple shared by two numbers.
Venn diagram – A diagram showing common and distinct factors/multiples.
Simplify – Making calculations easier by reducing numbers using factors.

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

This list continues and doesn't end

$3x$, $6x$, $9x$ …

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5

Not an integer

Factors

Arrays can help represent factors

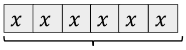
5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

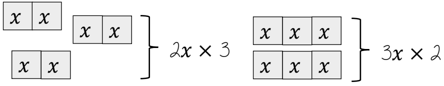
The number itself is always a factor

Factors and expressions



$6x \times 1$ OR $6 \times x$

Factors of $6x$
6, x, 1, $6x$, $2x$, 3, $3x$, 2



Prime numbers

- Integer
- Only has 2 factors
- and itself

2

The first prime number
The only even prime number

Learn or how-to quick recall…

2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

Square and triangular numbers

Square numbers

odd even odd

Representations are useful to understand a square number n^2

1, 4, 9, 16, 25, 36, 49, 64 …

Triangular numbers

Representations are useful – an extra counter is added to each new row

Odd two consecutive triangular numbers and get a square number

1, 3, 6, 10, 15, 21, 28, 36, 45…

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM

Common multiples are multiples two or more numbers share

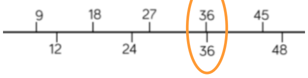
LCM – Lowest common multiple

LCM of 9 and 12

9	9, 18, 27, 36, 45, 54
12	12, 24, 36, 48, 60

LCM = 36

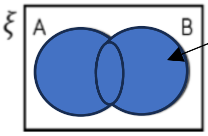
The first time their multiples match



Comparing fractions

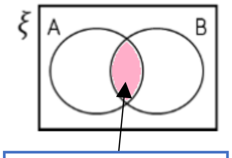
$\frac{3}{5}$ and $\frac{7}{10}$ ← Compare fractions using a LCM denominator → $\frac{6}{10}$ and $\frac{7}{10}$

Venn diagrams



The LCM is the numbers in both circles

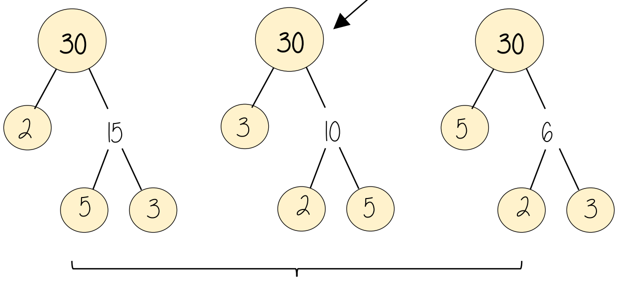
Remember: you must multiply the numbers together



The HCF would be the number in overlap

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5

Multiplication is commutative

Multiplication of prime factors

Using prime factors for predictions

e.g. 60 30 x 2 2 x 3 x 5 **x 2**
150 30 x 5 2 x 3 x 5 **x 5**

YEAR 9 — AUTUMN

N12 - PERCENTAGES

Sparx Maths

Percentage increase and decrease – M476, M533, U773, U671
Express a change as a percentage–U278 Find the original value after a percentage change –U286
Solve problems with percentages (non-calculator) – M437, M476, U554, U773
Solve problems with percentages (calculator) – M905, M533, U349, U671
Repeated percentage change – U332, U988Understand interest – U533
Simple interest – U533 Compound interest – U332

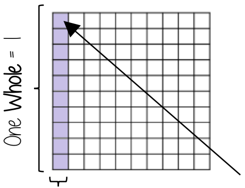
What do I need to be able to do?

Step 1 Percentage increase and decrease
Step 2 Express a change as a percentage
Step 3 Find the original value after a percentage change
Step 4 Solve problems with percentages (non-calculator)
Step 5 Solve problems with percentages (calculator)
Step 6 Repeated percentage change
Step 7 Understand interest
Step 8 Simple interest
Step 9 Compound interest

Keywords

Percent: parts per 100 – written using the % symbol.
Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
Fraction: a fraction represents how many parts of a whole value you have.
Equivalent: of equal value.
Reduce: to make smaller in value.
Growth: to increase/ to grow.
Integer: whole number, can be positive, negative or zero.
Invest: use money with the goal of it increasing in value over time (usually in a bank).
Multiplier: the number you are multiplying by.
Profit: the income take away any expenses/ costs.

FDP Equivalence

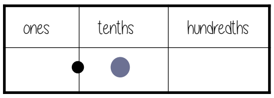


Percentage
100% = a whole = 100 hundredths

10 hundredths
10 out of 100
10%

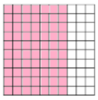
$$\frac{10}{100} = \frac{1}{10} = 0.10$$

One **hundredth**
(one whole split into 100 equal parts)



Converting FDP

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares
70 "hundredths" = 7 "tenths"
0.7



→ 70 hundredths = 70%

Using a **calculator**

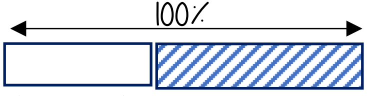
$\frac{■}{□}$ → S⇌D Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals
e.g $\frac{1}{3}$ = 0.3333333
= $0.\dot{3}$
The dot above the 3

Percentage Increase/ Decrease

Decrease

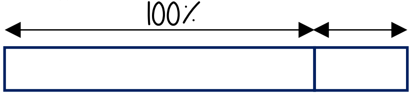


42%

Decrease by 58%

1.00 – 0.58 = 0.42 ← **Multiplier** Less than 1

Increase

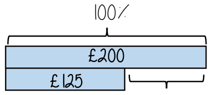


Increase by 12%

100% + 12% = 112%
1.00 + 0.12 = 1.12 ← **Multiplier** More than 1

Percentage change

I bought a phone for £200.
A year later sold it for £125.



All values of change compare to the ORIGINAL value

Percentage loss

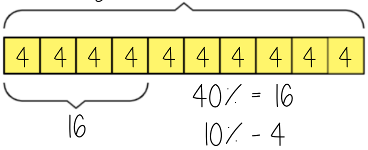
$$\frac{75}{200} \times 100 = 37.5\%$$

$$\frac{Difference\ in\ values}{Original\ value} \times 100$$

Reverse Percentages

40% of my number is 16. What am I thinking of?

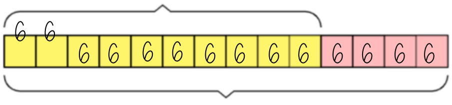
Original Number (100%)



40% = 16
10% - 4
100% = 40

140% of my number is 84. What is the original number?

Original Number (100%)



140% = 84
10% - 6
100% = 60

Try to scale down to 10% or 1% and then scale back up to 100%

Simple Interest

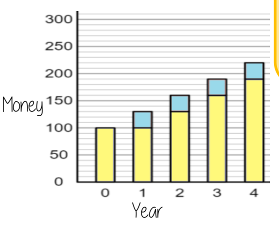
For each year of investment the interest remains the same

$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account.

e.g Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$



This account earned **£120** interest.
At the end of year 4 they have **£220**

Compound Interest

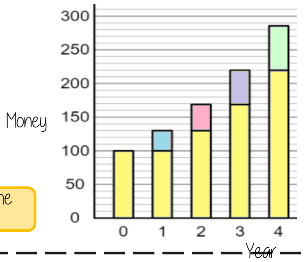
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g Invest £100 at 30% compound interest for 4 years

$$100 \times 1.30^4 = £285.61$$

This account has **£285.61** in total at the end of the 4 years.



YEAR 9 — AUTUMN

G7 - AREA AND VOLUME

Sparx Maths

Nets – M518 Area of a 2-D shape – M390, M610, M705, M269
Area and circumference of a circle – M231, M169
Surface area of cubes and cuboids – M534 Surface area of a triangular prism (E) – M661
Surface area of a cylinder (E) – U464 Volume of a prism – M722
Volume of a cylinder – U915 Volume of cones, pyramids and spheres (E) – U116, U484, U617
Convert metric units of area and volume (E) – M728, M465, U248, U468

What do I need to be able to do?

- **Step 1** Nets
- **Step 2** Area of a 2-D shape
- **Step 3** Area and circumference of a circle
- **Step 4** Surface area of cubes and cuboids
- **Step 5** Surface area of a triangular prism (E)
- **Step 6** Surface area of a cylinder (E)
- **Step 7** Volume of a prism
- **Step 8** Volume of a cylinder
- **Step 9** Volume of cones, pyramids and spheres (E)
- **Step 10** Convert metric units of area and volume (E)

Keywords

2D: two dimensions to the shape e.g. length and width

3D: three dimensions to the shape e.g. length, width and height

Vertex: a point where two or more lines segments meet

Edge a line on the boundary joining two vertex

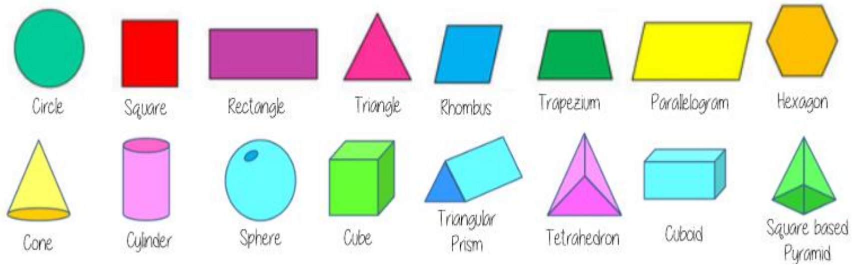
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

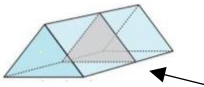
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

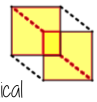
Name 2D & 3D shapes



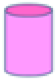
Recognise prisms

A solid object with two identical ends and flat sides



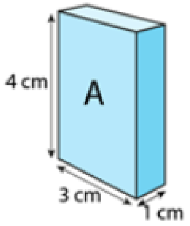


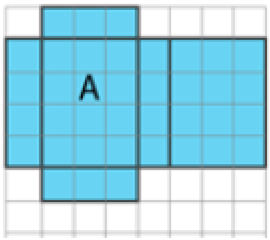
The cross section will also be identical to the end faces.



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

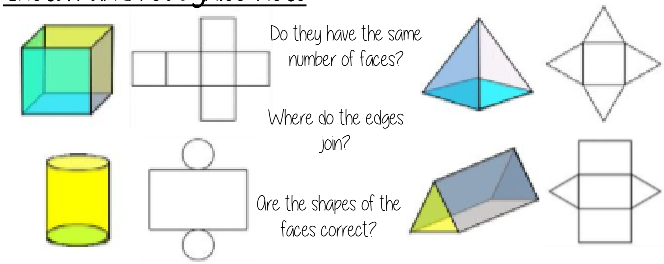




1cm grids help to draw accurately

Visualise the folding of the net
Will it make the cuboid with all sides touching

Sketch and recognise nets

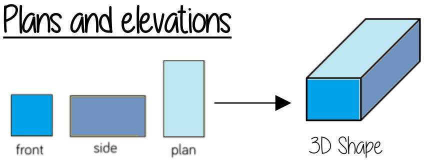


Do they have the same number of faces?

Where do the edges join?

Are the shapes of the faces correct?

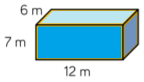
Plans and elevations

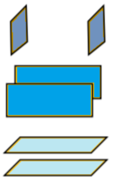


The direction you are considering the shape from determines the front and side views

Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area

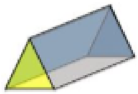




Sides	6 x 7 6 x 7
Front and back	12 x 7 12 x 7
Top and Bottom	12 x 6 12 x 6

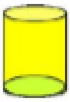
Sum of all sides is surface area

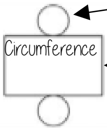
For cubes and cuboids you can also find one of each face and double it



For other shapes = not all the sides are the same, so calculate the individually

Surface area - cylinders





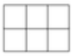
The area of the circle
π x radius^2

The width of this face is the same as the circumference
π x diameter x height

2 x π x radius^2 + π x diameter x height

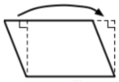
Area of 2D shapes

Rectangle
Base x Height

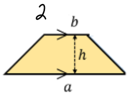


Triangle
½ x Base x Perpendicular height

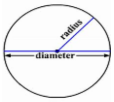
Parallelogram/ Rhombus
Base x Perpendicular height



Area of a trapezium
$\frac{(a+b) \times h}{2}$



Area of a circle
π x radius^2



Volumes

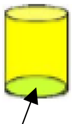
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

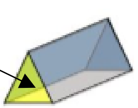
Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative





Cross section

Cross section

Prisms and cylinders = area cross section x height

Height can also be described as depth

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

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•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 9 HALF TERM 2 (Autumn 2) :

A9 - EQUATIONS, INEQUALITIES AND FORMULAE

N13 - FRACTIONS

R4 - RATES

N14 - STANDARD FORM

YEAR 9 — AUTUMN
A9 - EQUATIONS, INEQUALITIES AND FORMULAE

Solve equations and inequalities – M707, M118, U325, U759
Solve equations and inequalities with brackets – M902, U337 Inequalities with negative numbers (E) – U738
Solve equations and inequalities with unknowns on both sides – M554, U870
Solve problems with equations and inequalities – M957, U599
Substitute into formulae and equations – M208, M979, U585
Change the subject of a formula (one-step) – U675
Change the subject of a formula (two-step) – U181

What do I need to be able to do?

Step 1 Solve equations and inequalities
Step 2 Solve equations and inequalities with brackets
Step 3 Inequalities with negative numbers (E)
Step 4 Solve equations and inequalities with unknowns on both sides
Step 5 Solve problems with equations and inequalities
Step 6 Substitute into formulae and equations
Step 7 Change the subject of a formula (one-step)
Step 8 Change the subject of a formula (two-step)
Step 9 Change the subject of complex formula (E)

Keywords

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

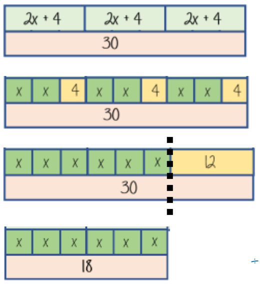
Rearrange: Change the order

Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

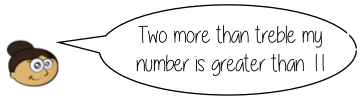
$-12 \qquad -12$

$6x = 18$

$\div 6 \qquad \div 6$

$\underline{x = 3}$

Form and solve inequalities



Find the possible range of values

$3x + 2 > 11$

Solve

$x \longleftarrow \div 3 \longleftarrow -2 \longleftarrow 11$

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \qquad +3x$

$2 > 17 + 3x$

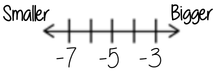
$-17 \qquad -17$

$-15 > 3x$

$\div 3 \qquad \div 3$

$\underline{-5 > x}$

x is true for any value smaller than -5



✓ CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \qquad -2$

$-3x > 15$

$\div -3 \qquad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

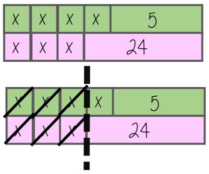
$4x + 5 = 3x + 24$

$-3x \qquad -3x$

$x + 5 = 24$

$-5 \qquad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

✓ -20 IS smaller than -18

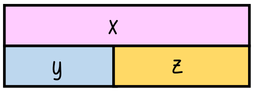
Formulae and Equations

Formulae – all expressed in symbols

Substitute in values

Equations – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

$y \longrightarrow +z \longrightarrow x$

$y \longleftarrow -z \longleftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)	In a formula (make x the subject)
$4x - 3 = 9$	$xy - s = a$
$+3 \qquad +3$	$+s \qquad +s$
$4x = 12$	$xy = a + s$
$\div 4 \qquad \div 4$	$\div y \qquad \div y$
$\underline{x = 3}$	$x = \frac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$ Gradient $= \frac{4}{2} = 2$

YEAR 9 — AUTUMN

N13 - FRACTIONS

Sparx Maths

Add and subtract fractions – M835, M931
Multiply and divide fractions – M157, M110, M197, M265
Fraction of an amount – M695, M684

What do I need to be able to do?

Step 1 Add and subtract fractions

Step 2 Multiply and divide fractions

Step 3 Fraction of an amount

Keywords

Numerator : the number above the line on a fraction. The top number. Represents how many parts are taken

Denominator: the number below the line on a fraction. The number represent the total number of parts

Equivalent: of equal value

Mixed numbers: a number with an integer and a proper fraction

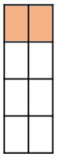
Improper fractions: a fraction with a bigger numerator than denominator

Substitute: replace a variable with a numerical value

Place value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

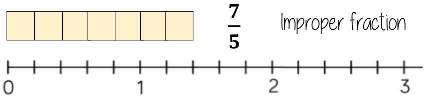
Representing Fractions

$\frac{1}{4}$ is represented in all the images

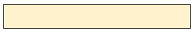


$1 \div 4$

Mixed numbers and fractions



$\frac{7}{5}$ Improper fraction



$1\frac{2}{5}$ Mixed number

In this model 5 parts make up a whole

Fractions can be bigger than a whole

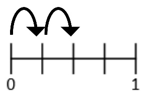
Add/Subtract unit fractions

Same denominator

$\frac{1}{12} + \frac{1}{12} - \frac{1}{12}$

$= \frac{2}{12}$

$\frac{1}{4} + \frac{1}{4}$



$= \frac{2}{4}$

With the same denominator ONLY the numerator is added or subtracted

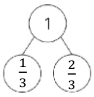
Add/Subtract fractions

Same denominator

$\frac{2}{7} + \frac{3}{7}$

$= \frac{5}{7}$

Sequences



$\frac{1}{3}, 1, 1\frac{2}{3}, 2\frac{1}{3}, 3, \ldots$

$+\frac{2}{3}$ $+\frac{2}{3}$

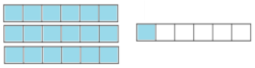
Represent this on a number line to help

Add/Subtract from integers

$1 - \frac{2}{6}$

$= \frac{4}{6}$

$3 + \frac{1}{6}$



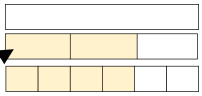
$= 3\frac{1}{6}$

The denominator indicates the number of parts a whole is made up of

Equivalent fractions

Numerator and denominator have the same multiplier

$\frac{2}{3} = \frac{4}{6}$



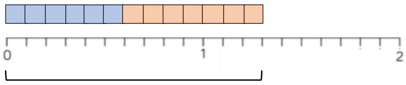
$\frac{1}{3} = \frac{2}{6}$

Add/Subtraction fractions (common multiples)

$\frac{3}{5} + \frac{7}{10}$

$\frac{6}{10} + \frac{7}{10}$

Addition/Subtraction needs a common denominator



$\frac{13}{10}$

Add/Subtraction any fractions

$\frac{4}{5} - \frac{2}{3}$

$\frac{12}{15} - \frac{10}{15}$

$= \frac{2}{15}$

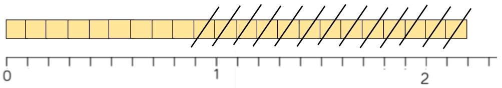
Use equivalent fractions to find a common multiple for both denominators

Add/Subtraction fractions (improper and mixed)

$2\frac{1}{5} - 1\frac{3}{10}$

$2\frac{2}{10}$

$\frac{22}{10} - \frac{13}{10}$



$= \frac{9}{10}$

- Convert to an improper fraction
- Calculate with common denominator

Partitioning method

$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$

Quick Multiplying and Cancelling down

The 3 and the 9 have a common factor and can be simplified

$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{4}{\cancel{9}\,3} \quad \frac{1 \times 4}{5 \times 3} = \frac{4}{15}$

Quick Solving
Multiply the numerators
Multiply the denominators

Dividing any fractions

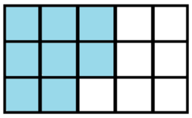
Remember to use reciprocals

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

YEAR 9 — AUTUMN

R4 - RATES

Sparx Maths

Speed, distance and time – U151, U902, U585, U144
Distance-time graphs – U403, U914, U462, U966
Solve flow problems and their graphs – U652, U862, U896
Rates of change and their units – U256, U151, U527, U910
Convert compound units (E) – U388, U248, U468

What do I need to be able to do?

Step 1 Speed, distance and time
Step 2 Distance-time graphs
Step 3 Solve flow problems and their graphs
Step 4 Rates of change and their units
Step 5 Convert compound units (E)

Keywords

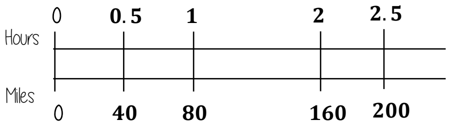
Convert: change
Mass: a measure of how much matter is in an object. Commonly measured by weight.
Origin: the coordinate (0, 0)
Volume: the amount of 3D space a shape takes up
Substitute: putting numbers where letters are – replacing numbers into a formula

Speed, Distance, Time

"per" for every
e.g. 80 miles per hour (mph)
Travel 80 miles every hour

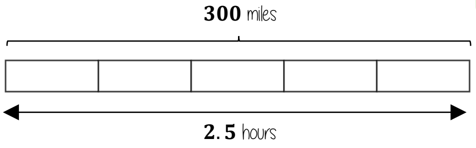
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.

Bar models can help to calculate mph



Each part is half an hour
Each part is 60 miles

Speed, Distance, Time

minutes ÷ 60 hours
hours × 60 minutes

Before calculations – make sure you are working in the same units as the speed

Learn or learn how to rearrange the formular for speed, distance and time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Substitute in the variables given

$$\text{distance} = \text{speed} \times \text{time}$$

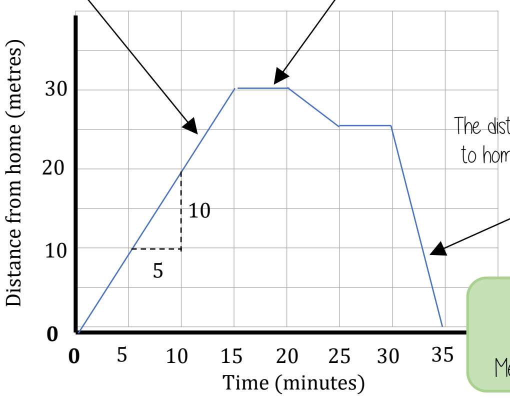
Distance – Time graphs

Gradient = speed

The steeper a gradient the faster the speed

$\frac{10}{5} = 2$ metres per min

Horizontal lines represent staying still



The distance coming closer to home shows the return journey

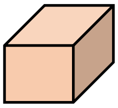
Units are important. Meters per minute

Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

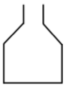
$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$

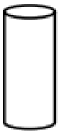


volume of prism = Area of cross section × Depth

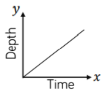
Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important. Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles **per** hour
Exchange rates: euros **per** pounds
Density: mass **per** volume

metres ÷ 1000 kilometres
kilometres × 1000 metres

YEAR 9 — AUTUMN

N14 - STANDARD FORM

Numbers in standard form – M719, U330
Compare and order numbers in standard form M719, U330
Multiply and divide numbers in standard form – U264
Add and subtract numbers in standard form – U290

What do I need to be able to do?

Step 1 Numbers in standard form
Step 2 Compare and order numbers in standard form
Step 3 Multiply and divide numbers in standard form
Step 4 Add and subtract numbers in standard form

Keywords

Standard (index) Form: A system of writing very big or very small numbers
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.

Positive powers of 10

1 billion – 1 000 000 000
$10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^9$

Addition rule for indices $10^a \times 10^b = 10^{a+b}$

Subtraction rule for indices $10^a \div 10^b = 10^{a-b}$

Standard form with numbers > 1

$A \times 10^n$

Any number between 1 and less than 10

Any integer

Example

3.2×10^4
$= 3.2 \times 10 \times 10 \times 10 \times 10$
$= 32000$

Non-example

0.8×10^4

$5.3 \times 10^{0.7}$

Negative powers of 10

0.001

$1 \times \frac{1}{1000}$

1×10^{-3}

10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^1	10^0	10^{-1}	10^{-2}	10^{-3}
0	0	0	0	1

Any value to the power 0 always = 1

Negative powers do not indicate negative solutions

Numbers between 0 and 1

0.054
$= 5.4 \times 10^{-2}$

1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
10^0	10^{-1}	10^{-2}	10^{-3}
0	0	5	4

A negative power does not mean a negative answer – it means a number closer to 0

Order numbers in standard form

10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}

6.4×10^{-2}	2.4×10^2	3.3×10^0	1.3×10^{-1}
0.064	240	1	0.13

Look at the power first will the number be = > or < than 1

Use a place value grid to compare the numbers for ordering

Mental calculations

$6.4 \times 10^2 \times 1000$ Not in Standard Form
$= 6.4 \times 10^2 \times 10^3$ Use addition for indices rule
$= 6.4 \times 10^5$

$(2 \times 10^3) \div 4$ Divide the values
$= (2 \div 4) \times 10^3$
$= 0.5 \times 10^3$

$\textcircled{8} \times 10^5 \times \textcircled{3}$
$= 24 \times 10^5$ Not in Standard Form
$= 2.4 \times 10^1 \times 10^5$ Use addition for indices rule
$= 2.4 \times 10^6$

Remember the layout for standard form

$A \times 10^n$

Any number between 1 and less than 10

Any integer

Addition and Subtraction

Tip: Convert into ordinary numbers first and back to standard from at the end

$$6 \times 10^5 + 8 \times 10^5$$

Method 1

$= 600000 + 800000$
$= 1400000$
$= 1.4 \times 10^5$

More robust method
Less room for misconceptions
Easier to do calculations with negative indices
Can use for different powers

Method 2

$= (6 + 8) \times 10^5$
$= 14 \times 10^5$
$= 1.4 \times 10^1 \times 10^5$
$= 1.4 \times 10^5$

This is not the final answer

Only works if the powers are the same

Multiplication and division

$\frac{1.5 \times 10^5}{0.3 \times 10^3}$

Division questions can look like this

For multiplication and division you can look at the values for A and the powers of 10 as two separate calculations

$(\textcircled{1.5} \times 10^5) \div (\textcircled{0.3} \times 10^3)$

$\textcircled{1.5 \div 0.3} \times 10^5 \div 10^3$

$= 5 \times 10^2$

Revisit addition and subtraction laws for indices – they are needed for the calculations

Addition law for indices
$a^m \times a^n = a^{m+n}$

Subtraction law for indices
$a^m \div a^n = a^{m-n}$

Using a calculator

$1.4 \times 10^5 \quad \times \quad 3.9 \times 10^3$

Use a calculator to work out this question to a suitable degree of accuracy.

Input 1.4 and press **x10ˣ** Then press 5 (for the power)
Press **×**
Input 3.9 and press **x10ˣ** Then press 3 (for the power)
Press **=**

This gives you the solution

Click calculator for video tutorial

To put into standard form and a suitable degree of accuracy

Press **SHIFT** **SETUP** and then press 7 for sci mode.

Choose a degree of accuracy so in most cases press 2

Answer: 5.5×10^8

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

- Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
- Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

- Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
- Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
- Apply reason to all that they do, determined to achieve their goals.

AMBITION:

- Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
- Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
- Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
- Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 9 HALF TERM 3 (SPRING 1) :

N15 - MATHS AND MONEY

A10 - STRAIGHT LINE GRAPHS

R5 - RATIO AND PROPORTION

YEAR 9 — SPRING

N15 - MATHS AND MONEY

Sparx Maths

What do I need to be able to do?

- **Step** 1 Understand a bank account
- **Step** 2 Spending
- **Step** 3 Ways to pay
- **Step** 4 Ways to save
- **Step** 5 Jobs and pay
- **Step** 6 Investing
- **Step** 7 Borrowing (buying a house)
- **Step** 8 Running a house or a business
- **Step** 9 Budgeting
- **Step** 10 Borrowing (loans)
- **Step** 11 Spending overseas
- **Step** 12 Insurance

Keywords

Credit: money being placed into a bank account
Debit: money that leaves a bank account
Balance: the amount of money in a bank account
Expense: a cost/ outgoing
Deposit: an initial payment (often a way of securing an item you will later pay for)
Multiplier: a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)
Per Annum: each year
Currency: the type of money a country uses.
Unitary: one – the cost of one.

Bills and Bank Statements

Bills – tell you the amount items cost and can show how much money you need to pay.

Some can include a total

Look for different units (Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19th Sept	Salary	£1500		£1500
19th Sept	Mortgage		£600	£900
25th Setp	Bday Money	£15		£915

Simple Interest

For each year of investment the interest remains the same

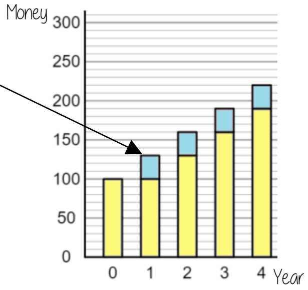
$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account.

e.g. Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = £120$$

This account earned **£120** interest.
At the end of year 4 they have **£220**



Compound Interest

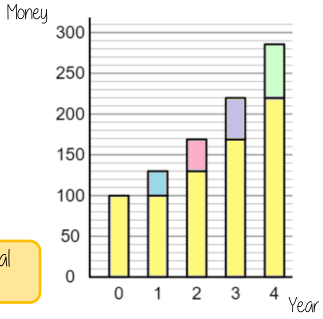
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g. Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = £285.61$$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

Wages and Taxes

Salaries fall into tax brackets – which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:
Time and a half – means 1.5 times their hourly rate
Double – 2 times their hourly rate

Exchange Rates

$$\begin{tikzcd}
£1 \arrow[r, "\times 1.4"] \arrow[d, "\times 200"'] & \$1.40 \arrow[d, "\times 200"] \\
£200 \arrow[r, "\times 1.4"'] & £280
\end{tikzcd}$$

When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies		
United Kingdom	£	Pounds
United States of America	\$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges £1	5 cupcakes £1.20

$4 = £1.00$ ÷ 2
$2 = £0.50$ ÷ 2
$1 = £0.25$

$5 = £1.20$ ÷ 5
$1 = £0.20$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

YEAR 9 — SPRING

A10 - STRAIGHT LINE GRAPHS

Sparx Maths

Lines, parallel to the axes, y = x and y = -x – M797
Explore gradients – U315, U741 Explore intercepts – U669
y = mx + c – U315, U74 lRearrange equations to the form y = mx + c (E) – U315, U669
Find the equation of a line from a graph – U315, U848, U477
Interpret gradient and intercepts of real-life graphs – U652, U862, U669
Graph inequalities (E) – U747

What do I need to be able to do?

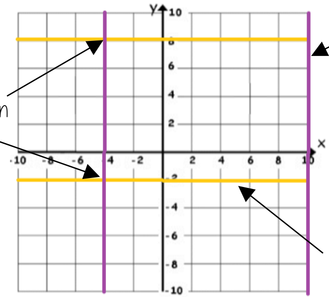
Step 1 Lines, parallel to the axes, y=x and y=-x
Step 2 Explore gradients
Step 3 Explore intercepts
Step 4 y=mx+c
Step 5 Rearrange equations to the form y=mx+c
Step 6 Find the equation of a line from a graph
Step 7 Interpret gradient and intercepts of real-life graphs
Step 8 Graph inequalities (E)

Keywords

Gradient: the steepness of a line
Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.
Parallel: two lines that never meet with the same gradient.
Co-ordinate: a set of values that show an exact position on a graph.
Linear: linear graphs (straight line) — linear common difference by addition/ subtraction
Asymptote: a straight line that a graph will never meet.
Reciprocal: a pair of numbers that multiply together to give 1.
Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes

R



Intersection points

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

Plotting y = mx + c graphs

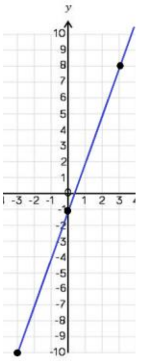
R

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

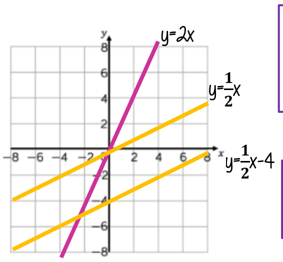
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

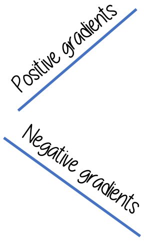
$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



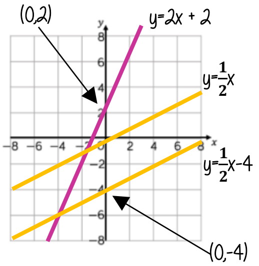
The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient



Compare Intercepts

$y = mx + c$ ← The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**.

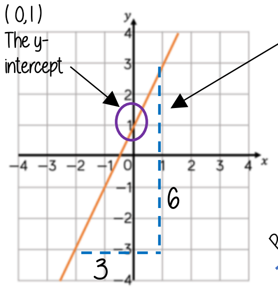
The equation of a line can be rearranged. Eg:

y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

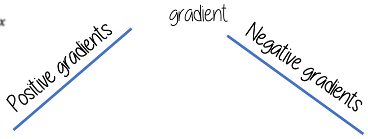
Find the equation from a graph



The Gradient
$\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient



Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour.
Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge.
The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost.
The gradient shows the price per pen.

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

YEAR 9 — SPRING

R5 - RATIO AND PROPORTION

Sparx Maths

Direct proportion – U721
Direct proportion and conversion graphs – U721, U640, U652
Inverse proportion – U357, U364
Inverse proportion graphs (E) – U238
Ratio problems (whole or part given) – U577, U753
Solve problems with ratio and algebra (E) – U676, U865

What do I need to be able to do?

Step 1 Direct proportion
Step 2 Direct proportion and conversion graphs
Step 3 Inverse proportion
Step 4 Inverse proportion graphs (E)
Step 5 Ratio problems (whole or part given)
Step 6 Solve problems with ratio and algebra (E)

Keywords

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

4 cans of pop = £2.40
x 0.5 ↓ ↓ x 0.5
2 cans of pop = £1.20

This multiplier is the same In the same way that this would be for ratio

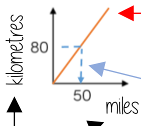
This is a multiplicative change

4 cans of pop = £2.40
x 3 ↓ ↓ x 3
12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first e.g. **1 can of pop = £0.60**

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2 × 4

T	1	2	8
G	40	20	5

× 2 ÷ 4

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

	Shop A	Shop B
	4 cans for £1.20	3 cans for 93p
	£1.20 ÷ 4	£0.93 ÷ 3
Cost per item	1 can is £0.30 Or 30p	1 can is £0.31 Or 31p

Shop A is the best value as it is 1p cheaper per can of pop

	Shop A	
	4 cans for £1.20	3 cans for 93p
	4 ÷ £1.20	3 ÷ £0.93
Cost per pound	£1 buys 3.333 cans of pop	£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound.

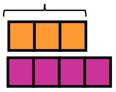
Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

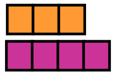
£350 ÷ 7 = £50

☐ = one part
= £50

Put back into the question

James: Lucy
x 50 3 : 4 x 50
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

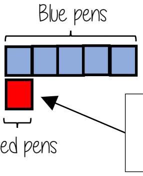
Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

☐ = one part
= 10 pens

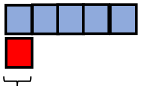


One unit
= 10 pens

Put back into the question

Blue : Red
x 10 5 : 1 x 10
50: 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are **50 Blue Pens**

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

- Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
- Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

- Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
- Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
- Apply reason to all that they do, determined to achieve their goals.

AMBITION:

- Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
- Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
- Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
- Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 9 HALF TERM 4 (Spring 2):

G8 - CONSTRUCTIONS AND CONGRUENCE

G9 - SIMILARITY

A11 - ALGEBRAIC MANIPULATION

YEAR 9 — SPRING

G8 - CONSTRUCTIONS AND CONGRUENCE

Draw and measure angles – M331 Construct and interpret scale drawings – M112 Construct triangles using ASA, SAS and SSS – U187 Construct an angle bisector – U787 Construct a perpendicular bisector – U245 Construct a perpendicular from or to a point – U245 Construct more complex polygons – U820 Identify congruent figures – U790 Congruent triangles – U866

What do I need to be able to do?

- **Step 1** Draw and measure angles
- **Step 2** Construct and interpret scale drawings
- **Step 3** Construct triangles using ASA, SAS and SSS
- **Step 4** Construct an angle bisector
- **Step 5** Construct a perpendicular bisector
- **Step 6** Construct a perpendicular from or to a point
- **Step 7** Construct more complex polygons
- **Step 8** Identify congruent figures
- **Step 9** Congruent triangles

Keywords

Protractor: piece of equipment used to measure and draw angles

Locus: set of points with a common property

Equidistant: the same distance

Discorectangle: (a stadium) – a rectangle with semi circles at either end

Perpendicular: lines that meet at 90°

Arc: part of a curve

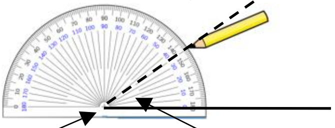
Bisector: a line that divides something into two equal parts

Congruent: the same shape and size

Draw and measure angles

Draw a 35° angle

Make a mark at 35° with a pencil And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

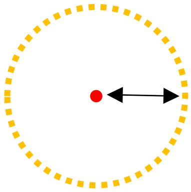
Image : Real life

1cm : 30cm

10cm : 300cm

× 10 × 10

Locus of a distance from a point

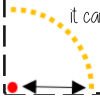


All points are equidistant (the same distance) from the fixed point in the middle.

Equipment needed

The radius is the distance from the fixed point

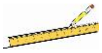
If the point is in the corner it can only make a quarter circle



Locus of a distance from a straight line

All points are equidistant (the same distance) from line

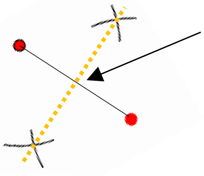
The ends of the line are fixed points



Equipment needed

The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points



Also a perpendicular bisector

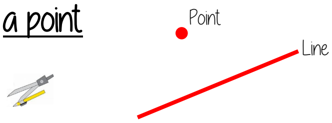
Because if the points are joined, this new line intersects it at a 90°

Keep the compass the same size and draw two arcs from each point

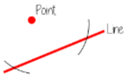
Join the intersections with a ruler.

All points on this line are equidistant from both points

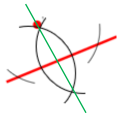
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



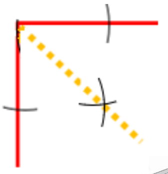
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector

This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

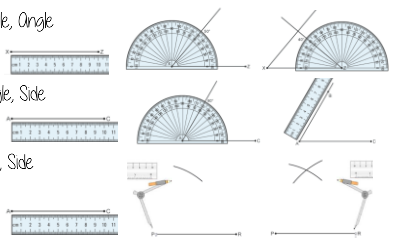
Constructing Triangles

Link to steps →

Side, Angle, Angle

Side, Angle, Side

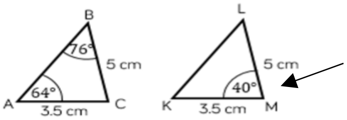
Side, Side, Side



Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 9 — SPRING

G9 - SIMILARITY

Sparx Maths

Recognise enlargement and similarity – U551
Work out unknown lengths and angles in similar shapes – U578
Solve problems with similar triangles (E) – U887
Ratio in right-angled triangles (E) – U605

What do I need to be able to do?

Step 1 Recognise enlargement and similarity

Step 2 Work out unknown lengths and angles in similar shapes

Step 3 Solve problems with similar triangles (E)

Step 4 Ratio in right-angled triangles (E)

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Centre of enlargement: the point the shape is enlarged from

Similar: when one shape can become another with a reflection, rotation, enlargement or translation.

Congruent: the same size and shape

Corresponding: items that appear in the same place in two similar situations

Parallel: straight lines that never meet (equal gradients)

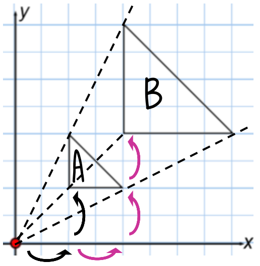
Positive scale factors

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

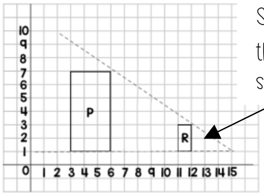
The distance from the point enlarges by 2



Fractional scale factors

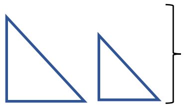
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



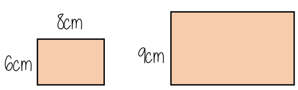
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°

Similar shapes

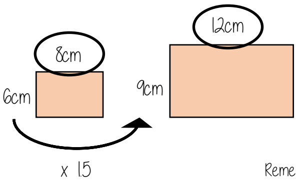


Scale Factor: Both sides on the bigger shape are 1.5 times bigger

Compare sides:	6 : 9	8 : 12
	2 : 3	2 : 3

Both sets of sides are in the same ratio

Information in similar shapes



Compare the equivalent side on both shapes

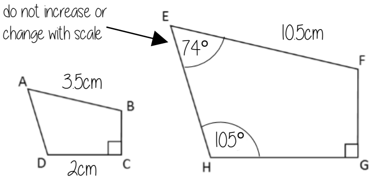
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

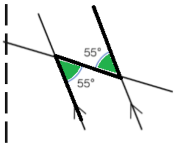
AB and EF are corresponding

Remember angles do not increase or change with scale



Angles in parallel lines

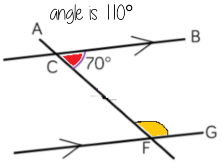
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

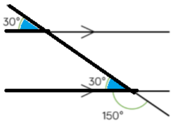
Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°



Corresponding angles

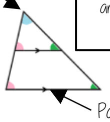
Because corresponding angles are equal the highlighted angles are the same size



As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Similar triangles

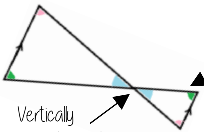
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines – all angles will be the same in both triangle

As all angles are the same this is similar – it only one pair of sides are needed to show equality

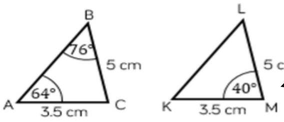


All the angles in both triangles are the same and so similar

Vertically opposite angles

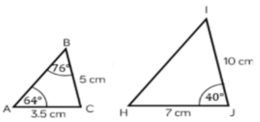
Congruence and Similarity

Congruent shapes are identical – all corresponding sides and angles are the same size



AĈB = KM̂L

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 9 — SPRING

A11 - ALGEBRAIC MANIPULATION

Sparx Maths

Expand single brackets and simplify – U179
Factorise into a single bracket – U365
Expand double brackets – U768 Use identities – U582
Factorise quadratic expressions (E) – U178, U858, U963
Solve quadratic equations (E) – U228, U960, U589, U665
Expand triple brackets (E) – U606

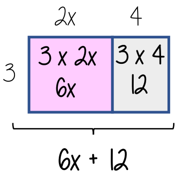
What do I need to be able to do?

Step 1 Expand single brackets and simplify
Step 2 Factorise into a single bracket
Step 3 Expand double brackets
Step 4 Use identities
Step 5 Factorise quadratic expressions (E)
Step 6 Solve quadratic equations (E)
Step 7 Expand triple brackets (E)

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: An equation where both sides have variables that cause the same answer includes ≡
Linear: an equation or function that is the equation of a straight line
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

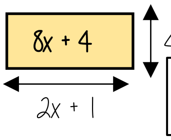
Multiply single brackets



Single: Where each term inside the bracket is multiplied by the term on the outside of the bracket.

$3(2x+4) = 6x + 12$

Factorise into a single bracket $8x + 4$



Try and make this the **highest common factor**

The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x+1)$

Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

Equation
A statement that two things are equal

Term

Identity
An equation where both sides have variables that cause the same answer includes ≡

Formula
A rule written with all mathematical symbols e.g area of a rectangle
A = b x h

Expanding double brackets

Double: Where each term in the first bracket is multiplied by all terms in the second bracket. A double bracket will be a quadratic equation.

$(p+2)(2p-1)$
$= 2p^2 + 4p - p - 2$
$= 2p^2 + 3p - 2$

$(p+2)^2 = (p+2)(p+2)$
$= p^2 + 2p + 2p + 4$
$= p^2 + 4p + 4$

Factorising Quadratics

Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Factorise:

Add to find the middle term 2+4

$x^2 + 6x + 8$

Multiply to find the end term ~~1 8~~ 2 4

$= (x+2)(x+4)$

Add to find the middle term -3 + 1

$x^2 - 2x - 3$

Multiply to find the end term 1 3

$= (x-3)(x+1)$

Expanding Triple brackets

Where every term inside each bracket is multiplied by every term all other brackets

$(p+3)(p-1)(p+4)$

$= (p^2 + 3p - p - 3)(p+4)$

$= (p^2 + 2p - 3)(p+4)$

$= p^3 + 4p^2 + 2p^2 + 8p - 3p - 12$

$= p^3 + 6p^2 + 5p - 12$

Solve when = 0

solve $3x + 4 = 0$
$-4 \quad -4$
So $3x = -4$
$\div 3 \quad x = \frac{-4}{3} \quad \div 3$

$\boxed{-4 + 4 = 0}$

Solve the equation $(2x+1)(1-x) = 0$

$\boxed{(2x+1)}\boxed{(1-x)} = 0$

Work with both solution separately

$2x + 1 = 0$
$-1 \quad -1$
$2x = -1$
$\div 2 \quad \div 2$
$x = \frac{-1}{2}$

$1 - x = 0$
$+x \quad +x$
$x = 1$

Therefore, the solutions are

Factorise and solve:

$x^2 + 4x - 5 = 0$

$(x-1)(x+5) = 0$

Either $x - 1 = 0$
$x = 1$

Or $x + 5 = 0$
$x = -5$

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the ‘why’ behind mathematical formulae.

We want our students to:

EXCELLENCE:

- Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
- Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

- Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
- Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
- Apply reason to all that they do, determined to achieve their goals.

AMBITION:

- Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
- Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
- Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
- Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 9 HALF TERM 5 (summer 1):

G10 - PYTHAGORAS’ THEOREM

A12 - NON- LINEAR GRAPHS

P2 - PROBABILITY

YEAR 9 — SUMMER

G10 - PYTHAGORAS' THEOREM

Sparx Maths

Solve equations with squares and square roots – U851
Identify the hypotenuse – U283 Determine whether a triangle is right-angled – U385
Pythagoras theorem (find the hypotenuse) – U385
Use Pythagoras theorem on coordinate axes – U828
Proofs of Pythagoras theorem (E) – U385 (conceptual), U828 (application)
Pythagoras theorem in 3-D shapes (E) – U541

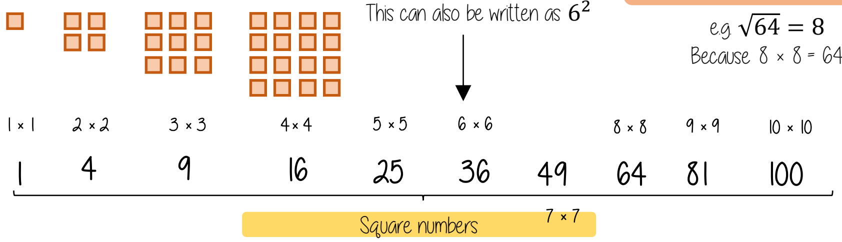
What do I need to be able to do?

Step 1 Solve equations with squares and square roots
Step 2 Identify the hypotenuse
Step 3 Determine whether a triangle is right-angled
Step 4 Pythagoras theorem (find the hypotenuse)'
Step 5 Pythagoras theorem (find any side)'
Step 6 Use Pythagoras theorem on coordinate axes'
Step 7 Proofs of Pythagoras theorem (E)'
Step 8 Pythagoras theorem in 3-D shapes (E)

Keywords

Square number: the output of a number multiplied by itself
Square root: a value that can be multiplied by itself to give a square number
Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.
Opposite: the side opposite the angle of interest
Adjacent: the side next to the angle of interest

Squares and square roots

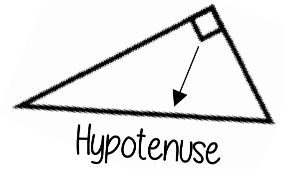


This can also be written as 6^2

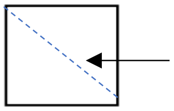
$\sqrt{}$ is the square root symbol

eg $\sqrt{64} = 8$
Because 8 × 8 = 64

Identify the hypotenuse

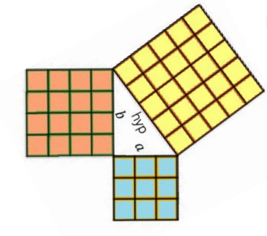


The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



a = 3 b = 4 c = 5

If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

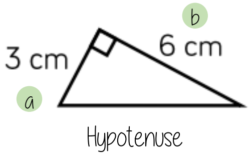
eg $a^2+b^2 = hypotenuse^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$3^2+6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

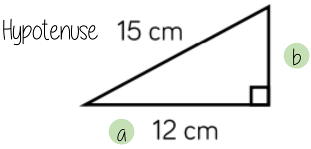
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$\sqrt{45} = \text{hypotenuse}$

$\mathbf{6.71}\boldsymbol{cm} = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$12^2+b^2 = 15^2$

1 Substitute in the values you are given

$$\underset{-144}{144} + b^2 = \underset{-144}{225}$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

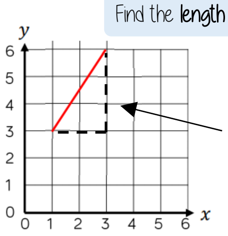
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$$\underbrace{a^2 + b^2}_{} = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes

YEAR 9 — SUMMER

A12 - NON- LINEAR GRAPHS

Sparx Maths

Substitute into quadratic expressions – U585
Draw simple quadratic graphs – U989 Draw more complex quadratic graphs – U989
Interpret quadratic graphs – U667 Interpret reciprocal and exponential graphs – U593, U229
Draw cubic graphs (E) – U980 Interpret cubic graphs (E) – U980
Interpret roots, intercepts and turning points (E) – U667, U769

What do I need to be able to do?

Step 1 Substitute into quadratic expressions
Step 2 Draw simple quadratic graphs
Step 3 Draw more complex quadratic graphs
Step 4 Interpret quadratic graphs
Step 5 Interpret reciprocal and exponential graphs
Step 6 Draw cubic graphs (E)
Step 7 Interpret cubic graphs (E)
Step 8 Interpret roots, intercepts and turning points (E)

Keywords

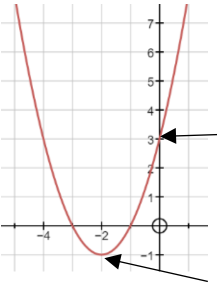
Quadratic: a curved graph with the highest power being 2. Square power.
Inequality: makes a non equal comparison between two numbers
Reciprocal: a reciprocal is 1 divided by the number
Cubic: a curved graph with the highest power being 3. Cubic power.
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

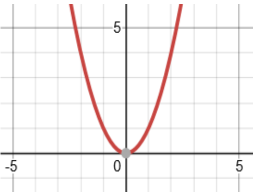
Coordinate pairs for plotting $(-3, 0)$

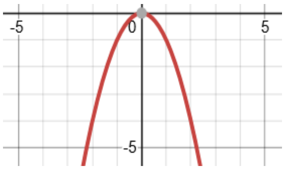
Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

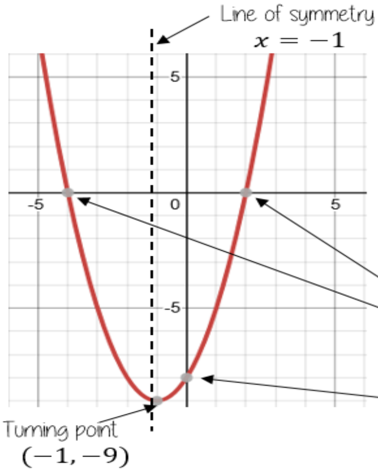
A quadratic graph will always be in the shape of a parabola.

$$y = x^2 \qquad y = -x^2$$





The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.



Examples

$$y = x^2 + 2x - 8$$

A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation.
This is dependant on how many times the graph crosses the x axis.

Roots $x = -4$
$x = 2$

y intercept = -8

Interpreting graphs

Interpret other graphs

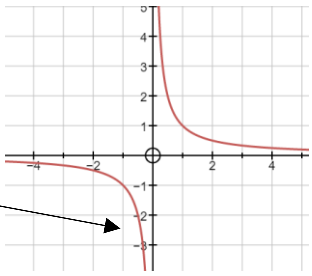
Cubic Graphs

$y = x^3 + 2x^2 - 2x + 1$

If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$$y = \frac{1}{x}$$

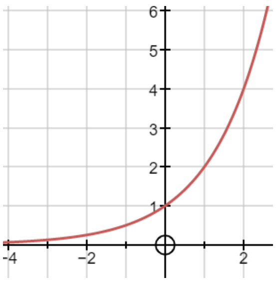


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x



YEAR 9 — SUMMER

P2 - PROBABILITY

Sparx Maths

Identify and represent sets – U748 Intersection of a set – U748
Union of a set – U748 Complement of a set (E) – U748
Probability of a single event – U510
Use diagrams to work out probabilities – U699, U806
Relative frequency – U580 Expected outcomes – U166
Independent events – U558 Probabilities from Venn diagrams – U699

What do I need to be able to do?

- **Step 1** Identify and represent sets
- **Step 2** Intersection of a set
- **Step 3** Union of a set
- **Step 4** Complement of a set (E)
- **Step 5** Probability of a single event
- **Step 6** Use diagrams to work out probabilities
- **Step 7** Relative frequency
- **Step 8** Expected outcomes
- **Step 9** Independent events
- **Step 10** Probabilities from Venn diagrams

Keywords

Probability: the chance that something will happen

Relative Frequency: how often something happens divided by the outcomes

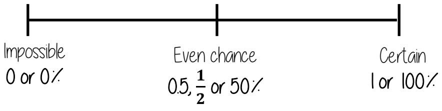
Independent: an event that is not effected by any other events.

Chance: the likelihood of a particular outcome.

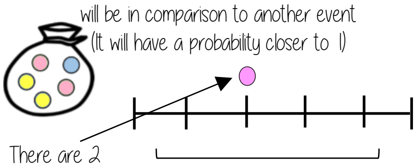
Event: the outcome of a probability – a set of possible outcomes.

Biased: a built in error that makes all values wrong by a certain amount.

The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Single event probability

Probability is always a value between 0 and 1

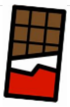
The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of **NOT** getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	



P(white chocolate) = 1 – 0.15 – 0.35
= 0.5

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times

$$0.3 \times 100 = 30$$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

On experiment is carried out 400 times.
Show that dark chocolate is expected to be selected 60 times

$$0.15 \times 400 = 60$$

Independent events

The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

Probability of event 1 × Probability of event 2

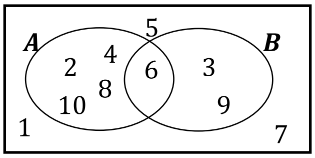
$P(5) = \frac{1}{6}$ $P(R) = \frac{1}{4}$

Find the probability of getting a 5 and a red

$$P(5\ and\ R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables



	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the ‘why’ behind mathematical formulae.

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•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 9 HALF TERM 6 (Summer 2):

G11 - TRANSFORMATIONS

A13 - SIMULTANEOUS EQUATIONS

G12 - TRIGONOMETRY

YEAR 9 — SUMMER
G11 — TRANSFORMATIONS
1 OF 2

Enlargement (positive scale factor) – U519 Enlargement from a point (positive scale factor) – U519 Enlargement (fractional scale factor) – U519
Enlargement (negative scale factor) (E) – U134 Describe an enlargement – U134
Rotation about a point – U696 Describe a rotation – U696
Translation – U196 Describe a translation – U196
Reflection – U799 Find the result of a series of transformations (E) – U766

What do I need to be able to do?

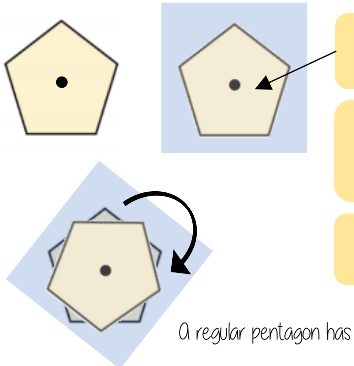
- **Step 1** Enlargement (positive scale factor)
- **Step 2** Enlargement from a point (positive scale factor)
- **Step 3** Enlargement (fractional scale factor)
- **Step 4** Enlargement (negative scale factor) (E)
- **Step 5** Describe an enlargement
- **Step 6** Rotation about a point
- **Step 7** Describe a rotation
- **Step 8** Translation
- **Step 9** Describe a translation
- **Step 10** Reflection
- **Step 11** Find the result of a series of transformations (E)

Keywords

Rotate: a rotation is a circular movement.
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.
Vertex: a point two edges meet.
Horizontal: from side to side
Vertical: from up to down
Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement

Rotational Symmetry

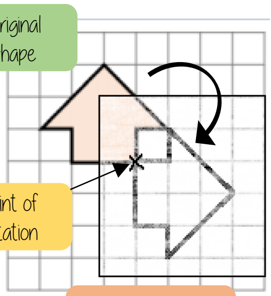
Tracing paper helps check rotational symmetry.



1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

a regular pentagon has rotational symmetry of order 5

Rotate from a point (in a shape)

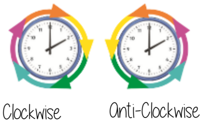


Original shape

Point of rotation

Image: 90° clockwise

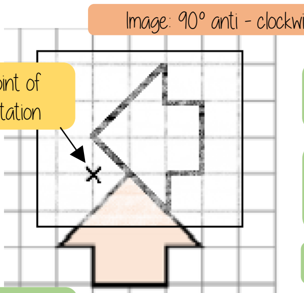
1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape



Rotate from a point (outside a shape)

Image: 90° anti - clockwise

Point of rotation



Original shape

1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

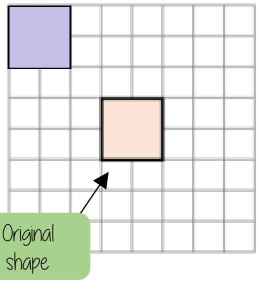
Translation and vector notation

Vector Notation → $\begin{pmatrix}1\\-2\end{pmatrix}$

How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

Translation $\begin{pmatrix}-3\\3\end{pmatrix}$



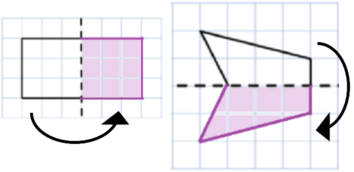
Original shape

Every vertex has been translated by the same amount

$\begin{pmatrix}-3\\3\end{pmatrix}$

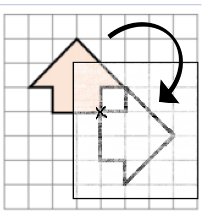
The image has been moved 3 squares to the left and 3 squares up

Compare rotations and reflections



Reflections are a mirror image of the original shape.

Information needed to perform a reflection:
- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:
- Point of rotation
- Direction of rotation
- Degrees of rotation

YEAR 9 — SUMMER
G11 — TRANSFORMATIONS
1 OF 2

Sparx Maths

Enlargement (positive scale factor) – U519 Enlargement from a point (positive scale factor) – U519 Enlargement (fractional scale factor) – U519
Enlargement (negative scale factor) (E) – U134 Describe an enlargement – U134
Rotation about a point – U696 Describe a rotation – U696
Translation – U196 Describe a translation – U196
Reflection – U799 Find the result of a series of transformations (E) – U766

What do I need to be able to do?

Step 1 Enlargement (positive scale factor)
Step 2 Enlargement from a point (positive scale factor)
Step 3 Enlargement (fractional scale factor)
Step 4 Enlargement (negative scale factor) (E)
Step 5 Describe an enlargement
Step 6 Rotation about a point
Step 7 Describe a rotation
Step 8 Translation
Step 9 Describe a translation
Step 10 Reflection
Step 11 Find the result of a series of transformations (E)

Keywords

Rotate: a rotation is a circular movement.
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.
Vertex: a point two edges meet.
Horizontal: from side to side
Vertical: from up to down
Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement

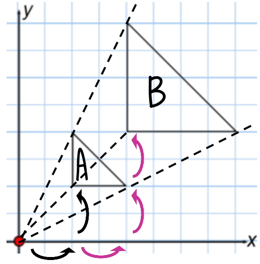
Positive scale factors

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

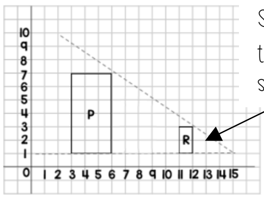
The distance from the point enlarges by 2



Fractional scale factors

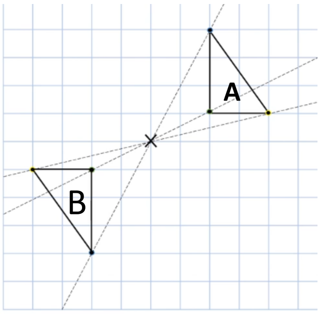
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



SF: $\frac{1}{3}$ - R is three times smaller than P

Negative scale factors

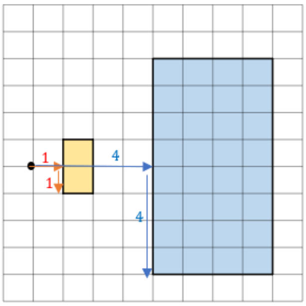


Enlarge shape A by SF -1 from the point shownto create shape B

A negative enlargement will flip each vertices

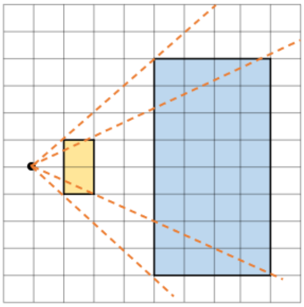
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

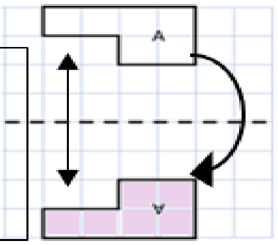
Rays method

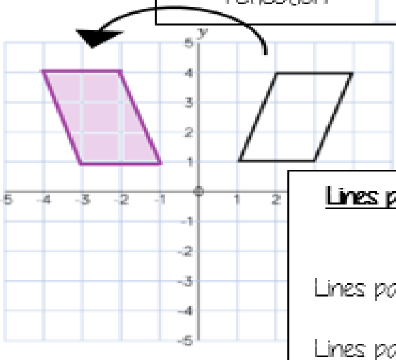


Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

Reflect horizontally/ vertically

All points need to be the same distance away from the line of reflection





Reflection in the line y axis — this is also a reflection in the line x=0

Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

Key Concepts

A **reflection** creates a mirror image of a shape on a coordinate graph. The mirror line is given by an equation eg. $y = 2, x = 2, y = x$. The shape does not change in size.

A **rotation** turns a shape on a coordinate grid from a given point. The shape does not change size but does change orientation.

An **enlargement** changes the size of an image using a scale factor from a given point.
A **positive scale factor** will increase the size of an image.
A **fractional scale factor** will reduce the size of an image.
A **negative scale factor** will place the image on the opposite side of the centre of enlargement, with the image inverted

A **translation** moves a shape on a coordinate grid. Vectors are used to instruct the movement:

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

x → Positive-Right, Negative - Left

y → Positive-Up, Negative - Down

YEAR 9 — SUMMER

A13 - SIMULTANEOUS EQUATIONS

Sparx Maths

Use one value to find another – U753 Introduction to simultaneous equations – U137
Solve simultaneous equations using graphs – U836
Solve simultaneous equations (no adjustments) – U760
Manipulating equations – U325, U870 Solve simultaneous equations (adjust one) – U760
Solve simultaneous equations (adjust both) (E) – U760 Solve simultaneous equations by substitution (E) – U757

What do I need to be able to do?

Step 1 Use one value to find another
Step 2 Introduction to simultaneous equations
Step 3 Solve simultaneous equations using graphs
Step 4 Solve simultaneous equations (no adjustments)
Step 5 Manipulating equations
Step 6 Solve simultaneous equations (adjust one)
Step 7 Solve simultaneous equations (adjust both) (E)
Step 8 Solve simultaneous equations by substitution (E)

Keywords

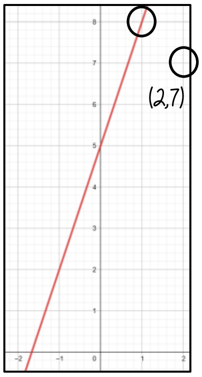
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position.
Intersection: the point two lines cross or meet.

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8



$y = 3x + 5$

$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5

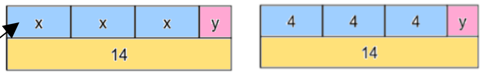
Substituting known variables

A line has the equation 3x + y = 14

Two different variables, two solutions

Stephanie knows the point x = 4 lies on that line. Find the value for y

$3x + y = 14$ $\qquad$ $3(4) + y = 14$



$x = 4$

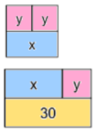
$$\begin{aligned} 12 + y &= 14 \\ -12 \quad & \quad -12 \\ y &= 2 \end{aligned}$$

Substituting in an expression

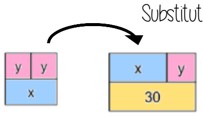
Substitute 2y in place of the x variable as they represent the same value

x = 2y

x + y = 30



Pair of simultaneous equations (two representations)

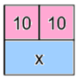


$x = 2y$ $\qquad$ $x + y = 30$

$3y = 30$

$$\begin{aligned} 3y &= 30 \\ \div 3 \quad & \quad \div 3 \\ y &= 10 \end{aligned}$$

$x = 2y$

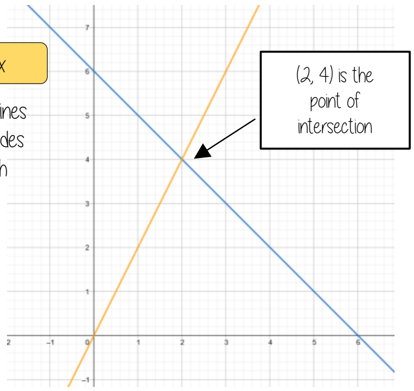


$x = 20$

Solve graphically

x + y = 6 $\qquad$ y = 2x

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations

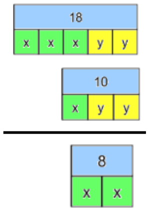


(2, 4) is the point of intersection

The solution that satisfies both equations is

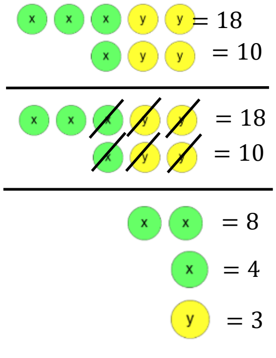
$x = 2$ and $y = 4$

Solve by subtraction



$$\begin{aligned} 3x + 2y &= 18 \\ -\quad x + 2y &= 10 \\ \hline 2x &= 8 \\ \div 2 \quad & \quad \div 2 \\ x &= 4 \\ \hline x + 2y &= 10 \\ (4) + 2y &= 10 \\ -4 \quad & \quad -4 \\ 2y &= 6 \\ \div 2 \quad & \quad \div 2 \\ y &= 3 \end{aligned}$$

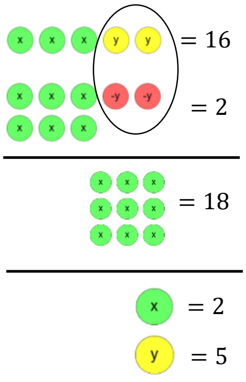
$x = 4$
$y = 3$



Solve by addition

$$\begin{aligned} 3x + 2y &= 16 \\ +\ 6x - 2y &= 2 \\ \hline 9x &= 18 \\ \div 9 \quad & \quad \div 9 \\ x &= 2 \\ \hline 3x + 2y &= 16 \\ 3(2) + 2(y) &= 16 \\ 6 + 2y &= 16 \\ -6 \quad & \quad -6 \\ 2y &= 10 \\ y &= 5 \end{aligned}$$

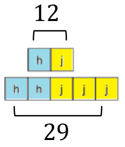
Addition makes zero pairs



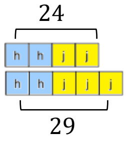
Solve by adjusting one

$h + j = 12$
$2h + 2j = 29$

No equivalent values



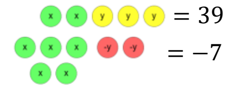
$2h + 2j = 24$
$2h + 2j = 29$



By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

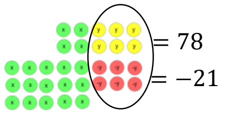
$2x + 3y = 39$
$5x - 2y = -7$



Use LCM to make equivalent x OR y values.
Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$
$15x - 6y = -21$

Now solve by addition



Addition makes zero pairs

YEAR 9 — SUMMER

G12 - TRIGONOMETRY

Sparx Maths

Identify hypotenuse, opposite and adjacent sides – U605
Use the tangent ratio to find unknown side lengths – U283
Use sine and cosine ratios to find unknown side lengths – U283
Use sine, cosine and tangent ratios to find unknown angles – U545
Choose the right method – U605, U283, U545 Key angles in right-angled triangles (E) – U627
Trigonometry in 3-D shapes (E) – U170, U541

What do I need to be able to do?

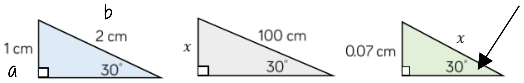
Step 1 Identify hypotenuse, opposite and adjacent sides
Step 2 Use the tangent ratio to find unknown side lengths
Step 3 Use sine and cosine ratios to find unknown side lengths
Step 4 Use sine, cosine and tangent ratios to find unknown angles
Step 5 Choose the right method
Step 6 Key angles in right-angled triangles (E)
Step 7 Trigonometry in 3-D shapes (E)

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Constant: a value that remains the same
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.
Inverse: function that has the opposite effect.
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

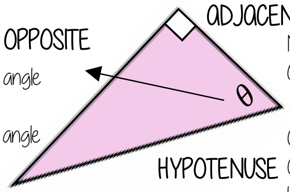
$$\frac{a}{b} = \frac{1}{2}$$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE
Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

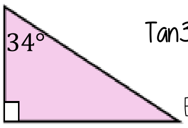
ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side
Always opposite the right angle
Useful to label this first

Tangent ratio: side lengths

$$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Substitute the values into the tangent formula



ADJACENT x cm
10cm OPPOSITE

$$\text{Tan}34 = \frac{10}{x}$$

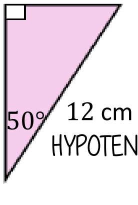
Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

$$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$$

Sin and Cos ratio: side lengths

OPPOSITE
x cm

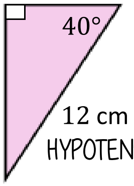


12 cm
HYPOTENUSE

$$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio

ADJACENT
x cm



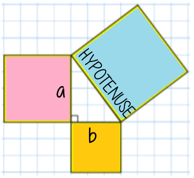
12 cm
HYPOTENUSE

$$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem Ⓡ



$$\text{Hypotenuse}^2 = a^2 + b^2$$

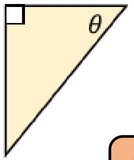
This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions

ADJACENT
3 cm



OPPOSITE
4 cm

Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$$\text{Tan}\theta = \frac{3}{4}$$

$$\theta = \text{Tan}^{-1}\frac{3}{4}$$

$\theta = 36.9°$

$$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$$

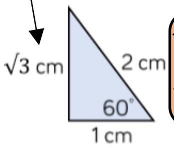
$$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$$

$$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$$

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements.

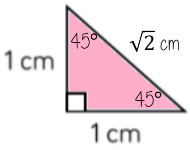
This side could be calculated using Pythagoras



$$\text{Tan}30 = \frac{1}{\sqrt{3}}$$
$$\text{Tan}60 = \sqrt{3}$$

$$\text{Cos}30 = \frac{\sqrt{3}}{2}$$
$$\text{Cos}60 = \frac{1}{2}$$

$$\text{Sin}30 = \frac{1}{2}$$
$$\text{Sin}60 = \frac{\sqrt{3}}{2}$$



$$\text{Tan}45 = 1$$

$$\text{Cos}45 = \frac{1}{\sqrt{2}}$$

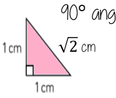
$$\text{Sin}45 = \frac{1}{\sqrt{2}}$$

Key angles 0° and 90°

Tan0 = 0

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



Sin0 = 0

Sin90 = 1

Cos0 = 1

Cos90 = 0