Crawshaw Academy

Knowledge Organisers

Year 10 Higher

A framework for effective home learning

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 1 (Autumn 1) :

A14 - ALGEBRAIC MANIPULATION

A15 - EQUATIONS, INEQUALITIES AND FORMULAE

A16 - QUADRATIC EXPRESSIONS AND EQUATIONS

YEAR 10H — AUTUMN

A14 - ALGEBRAIC MANIPULATION

Sparx Maths

Simplify expressions – U105
Laws of indices – U235, U694
Expand a single bracket – U179
Factorise into a single bracket – U365

What do I need to be able to do?

Step 1 Simplify expressions
Step 2 Laws of indices
Step 3 Expand a single bracket
Step 4 Factorise into a single bracket

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: An equation where both sides have variables that cause the same answer includes ≡
Linear: an equation or function that is the equation of a straight line

Like and unlike terms

Like terms are those whose variables are he same

Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.
It is used to identify equivalent expressions

Collecting like terms
Only **like terms** can be combined

$4x + 5b - 2x + 10b$

$(4x)\ [+5b]\ (-2x)\ [+10b]$

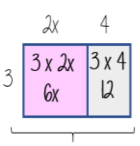
$2x + 15b$

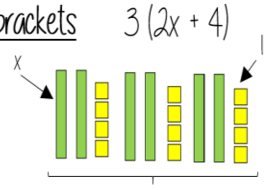
Common misconceptions

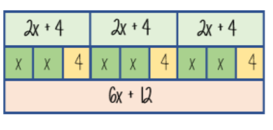
$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable x2 and x terms are unlike terms so can not be collected

Multiply single brackets $3(2x + 4)$

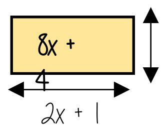






Different representations of $3(2x+4) = 6x + 12$

Factorise into a single bracket $8x + 4$



Try and make this the **highest common factor**

The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Algebraic numbers k is an odd number.

State whether each expression will be odd, even or could be either.

$k - 1$	$2k$	$2k + 1$	$3k$
Even	Even	Odd	Odd

Prove that the sum of two consecutive integers is odd.

Let n be an integer.
$n + 1$ is 1 greater than n
$n + n + 1 \equiv 2n + 1$
Even

Higher powers and roots

x^n

n – power (number of times multiplied by itself)

x – the base number.

Finding the nth root of any value $\sqrt[n]{x}$

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$
$= 9 \times 100$
$= 900$

Addition/ Subtraction Laws

$a^m \div a^n = a^{m-n}$

$a^m \times a^n = a^{m+n}$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$
$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated Use the addition law for indices

$(2^3)^4 = 2^{12}$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

FRACTIONAL INDICES

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$25^{\frac{1}{2}} = \sqrt{25} = 5$ (Square root)

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ (Cube root)

$25^{\frac{3}{2}} = (\sqrt{25})^3 = 5^3 = 125$

Remember that this is the same as $(25^{\frac{1}{2}})^3$

NEGATIVE FRACTIONAL INDICES

$a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$

$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}} = \frac{1}{5^3} = \frac{1}{125}$

Remember this is the same as $(25^{\frac{1}{2}})^3$

$8^{-\frac{1}{3}} = \frac{1}{8} = \frac{1}{2}$

Remember this means the cube root of 8!

YEAR 10H — AUTUMN

A15 - EQUATIONS, INEQUALITIES AND FORMULAE

Sparx Maths

Solve equations – U325 Solve fractional equations – U505
Solve equations with unknowns on both sides – U870
Understand inequalities – U509 Solve inequalities – U759, U738, U145
Represent solutions to inequalities using set notation (E) – U748
Change the subject of a known formula – U675
Change the subject of a formula – U181
Change the subject where the subject appears more than once (E) – U191

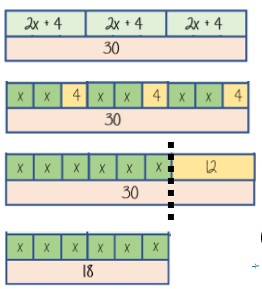
What do I need to be able to do?

- **Step 1** Solve equations
- **Step 2** Solve fractional equations
- **Step 3** Solve equations with unknowns on both sides
- **Step 4** Understand inequalities
- **Step 5** Solve inequalities
- **Step 6** Represent solutions to inequalities using set notation
- **Step 7** Change the subject of a known formula
- **Step 8** Change the subject of a simple formula
- **Step 9** Change the subject of a complex formula
- **Step 10** Change the subject where the subject appears more than once (E)

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: On equation where both sides have variables that cause the same answer includes ≡
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.
Fractional Equation: On equation that contains fractions,
Like Terms: Terms that have the same variable and exponent, which can be combined in an equation
Rearrange: To change the form of an equation or formula to make a different variable the subject.

Solve equations with brackets



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$
$-12 \quad -12$
$6x = 18$
$\div 6 \quad \div 6$
$\underline{x = 3}$

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow -3 \quad \leftarrow -2 \quad \leftarrow 11$

$x > 3$

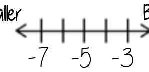
Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$
$+3x \quad +3x$
$2 > 17 + 3x$
$-17 \quad -17$
$-15 > 3x$
$\div 3 \quad \div 3$
$\underline{-5 > x}$

x is true for any value smaller than -5

✓ CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT



Method 2 Keep the negative x

$2 - 3x > 17$
$-2 \quad -2$
$-3x > 15$
$\div -3 \quad \div -3$
$\underline{x > -5}$

x is true for any value bigger than -5

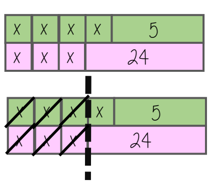
This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

$4x + 5 = 3x + 24$
$-3x \quad -3x$
$x + 5 = 24$
$-5 \quad -5$
$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$
$5x + 20 < 3x + 6$
$2x + 20 < 6$
$2x < -14$
$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$
$5(-4) < 3(-6)$
$-20 < -18$
✓ -20 IS smaller than -18

Formulae and Equations

Formulae – all expressed in symbols → **Substitute** in values → **Equations** – include numbers and can be solved

Rearranging Formulae (two step)

In an equation (find x)	In a formula (make x the subject)
$4x - 3 = 9$	$xy - s = a$
$+3 \quad +3$	$+s \quad +s$
$4x = 12$	$xy = a + s$
$\div 4 \quad \div 4$	$\div y \quad \div y$
$\underline{x = 3}$	$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g Find the gradient of the line $2y - 4x = 9$
Make y the subject first
$y = \dfrac{4x + 9}{2}$

Change the subject

Make v the subject of the formula

$$g = \frac{13(d - 3v)}{v}$$

get rid of the fraction first

$$vg = 13(d - 3v)$$
$$vg = 13d - 39v$$
$$vg + 39v = 13d$$

Factorise at this point to get the coefficient on its own

$$v(g + 39) = 13d$$

Rearrange to get the coefficient on its own

$$v = \frac{13d}{(g + 39)} \qquad v = \frac{13d}{(g + 39)}$$

YEAR 10H – AUTUMN

A16 - QUADRATIC EXPRESSIONS AND EQUATIONS

Sparx Maths

Expand double brackets – U768 Expand triple brackets – U606 Factorise quadratic expressions – U178
Factorise more complex quadratic expressions (E) – U858 Difference of two squares – U963
Solve quadratic equations equal to 0 – U228 Solve quadratic equations by factorisation – U228
Solve more complex quadratic equations by factorisation (E) – U960 Complete the square – U397
Solve quadratic equations by completing the square (E) – U589
Complete the square with more complex quadratic expressions (E) – U769
Solve quadratic equations using the quadratic formula – U665

What do I need to be able to do?

Step 1 Expand double brackets **Step 2** Expand triple brackets
Step 3 Factorise quadratic expressions
Step 4 Factorise more complex quadratic expressions (E)
Step 5 Difference of two squares
Step 6 Solve quadratic equations equal to 0
Step 7 Solve quadratic equations by factorisation
Step 8 Solve more complex quadratic equations by factorisation (E)
Step 9 Complete the square
Step 10 Solve quadratic equations by completing the square (E)
Step 11 Complete the square with more complex quadratic expressions (E)
Step 12 Solve quadratic equations using the quadratic formula

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Linear: an equation or function that is the equation of a straight line
Quadratic: a curved graph with the highest power being 2. Square power
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

Solving Quadratics

Quadratics are always in the form:

$$ax^2 + bx + c = 0$$

We can solve quadratic equations in 4 different ways

1. Factorising – put into brackets first

2. Completing the square

$$\left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2 = 0$$

3. Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Graphically

Factorising Quadratics to solve

Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Factorise:

Add to find the middle term 2+4

$x^2 + 6x + 8$

Multiply to find the end term ~~1 8~~ 2 4

$= (x + 2)(x + 4)$

Add to find the middle term -3 + 1

$x^2 - 2x - 3$

Multiply to find the end term 1 3

$= (x - 3)(x + 1)$

Factorise and solve:

$x^2 + 4x - 5 = 0 \qquad (x - 1)(x + 5) = 0$

Therefore the solutions are:

Either $x - 1 = 0$ Or $x + 5 = 0$

$x = 1 \qquad x = -5$

Factorising and solving with Coefficients

Solve: $6x^2 + 7x - 3 = 0$ (a, b, c)

Product = ac = −18 −2 & 9

Sum = b = 7

	$3x$	-1
$2x$	$6x^2$	$-2x$
$+3$	$+9x$	-3

$$(2x + 3)(3x - 1) = 0$$

$(2x + 3) = 0 \qquad (3x - 1) = 0$

$2x = -3 \qquad 3x = 1$

$x = -\frac{3}{2}$ And $x = \frac{1}{3}$

Completing the square

Completing the square is a method used to solve quadratic equations that will not factorise.

$x^2 + 4x - 15 = 0$

$(x + 2)^2$

$= (x + 2)(x + 2)$

$= x^2 + 4x + 4$

We don't want this extra.

$(x + 2)^2 - 4$

$-15 = 0$

$(x + 2)^2 - 4 - 15 = 0$

$(x + 2)^2 - 19 = 0$

We can solve quadratic using Completing the Square:

$x^2 - 4x + 1 = 0$

$(x - 2)^2 + 1 = 0$

$(x - 2)^2 - 4 + 1 = 0$

$(x - 2)^2 - 3 = 0$

$(x - 2)^2 = 3$

Rearrange to get x on its own.

$x - 2 = \pm\sqrt{3}$

$x = \pm\sqrt{3} + 2$

$x = \pm 1.7\ldots + 2$

Quadratic formula

When a quadratic doesn't factorise or is difficult to use completing the square method due to the value of its coefficients we use the quadratic formula:

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Watch out for double negatives with your 'b' value! If you are using a calculator remember to add brackets!

Solve: $x^2 - 5x + 2 = 0$

$a = +1 \qquad b = -5 \qquad c = +2$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - (4 \times (+1) \times (+2))}}{2 \times (1)}$$

$$x = \frac{5 \pm \sqrt{25 - 8}}{2}$$

$x = \frac{5 + \sqrt{17}}{2} \qquad x = \frac{5 - \sqrt{17}}{2}$

$x = 4.6 \qquad x = 0.4$

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Year 10 HALF TERM 2 (Autumn 2) :

N16 - PERCENTAGES

R6 - RATIO AND SCALE

N17 - WORK WITH FRACTIONS

YEAR 10H — AUTUMN

N16 - PERCENTAGES

Sparx Maths

Percentage of an amount – U554, U349 Percentage increase and decrease – U773, U671Repeated percentage change – U332 Express one number as a fraction or a percentage of another – U278 Express a change as a percentage – U278
Find the original value after a percentage change – U286
Simple interest – U533 Compound interest – U332
Choose appropriate methods to solve percentage problems – U721, U278, U286

What do I need to be able to do?

Step 1 Percentage of an amount
Step 2 Percentage increase and decrease
Step 3 Repeated percentage change
Step 4 Express one number as a fraction or a percentage of another
Step 5 Express a change as a percentage
Step 6 Find the original value after a percentage change
Step 7 Simple interest
Step 8 Compound interest
Step 9 Choose appropriate methods to solve percentage problems

Keywords

Percent: parts per 100 – written using the % symbol
Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
Fraction: a fraction represents how many parts of a whole value you have.
Equivalent: of equal value.
Reduce: to make smaller in value.
Growth: to increase/ to grow.
Integer: whole number, can be positive, negative or zero.
Invest: use money with the goal of it increasing in value over time (usually in a bank).
Multiplier: the number you are multiplying by.
Profit: the income take away any expenses/ costs.

Compare FDP

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares 70 "hundredths" = 7 "tenths" 0.7 → 70 hundredths = 70%

Using a calculator → S⇌D Convert to a decimal

This will give you the answer in the simplest form

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount

Find $\frac{3}{5}$ of £60

£60: £12 | £12 | £12 | £12 | £12 — £36

Remember $\frac{3}{5}$ = 60%

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5}$ = 60% = 0.6

60% of £60
= 0.6 x 60
= £36

Percentage increase/decrease

100%

42% — Decrease by 58%

100% - 58% = 42%

100 – 0.58 = 0.42 ← Multiplier Less than 1

100% | 12% — Increase by 12%

100% + 12% = 112%

100 + 0.12 = 1.12 ← Multiplier More than 1

Express as a percentage

27 per every 50 shaded: $\frac{27}{50}$ — 54 per every 100 shaded: $\frac{54}{100}$ — 54%

$\frac{13}{30}$ → ■/□ → $\frac{13}{30}$ → × 100

Can't use equivalence easily to find 'per hundred'

→ 43.3333...%
43%

Decimal percentages are still a percentage

Simple and compound interest

Simple Interest

James invests £2000 at 5% simple interest

100% | 5%: £2000 | £100

The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 — Original amount: £100
£100 | 10% £10 — Y1: £110
£110 | 10% £11 — Y2: £121
£121 | 10% £12.1 — Y3: £132.10

The multiplier 1.10 repeats each year

Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test.

Original x 0.6 = 24

60%: 24 — Total questions on test

24 ÷ 0.6 = 40 marks
10% = 6
100% = 40

A car sold for a profit £3000 with a profit of 20%. How much was the car originally?

100% | 20% — £3000

Original x 1.2 = 3000

120% = £3000
10% = £250
100% = £2500

Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 x 1.10 x 1.10 x 1.10

£100 x 1.10 [3]

Original amount — Repeated multiplier — Number of occurrences

Depreciation

Depreciation calculations use multipliers less than 1

Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

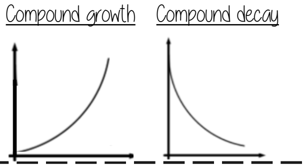
eg Increase of 10% then a reduction of 10%

x 1.10 x 0.9

x 0.99 The multiplier

Growth and decay

Compound growth — Compound decay



Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

YEAR 10H — AUTUMN

R6 - RATIO AND SCALE

Sparx Maths

Equivalent ratios – U753
Share in a ratio (given total, one part or difference) – U577
Link ratios and fractions – U176 Combine a set of ratios – U921
Share in a ratio (algebraically) – U676
Solve problems with ratio and algebra – U676
Ratios and scales – U112

What do I need to be able to do?

Step 1 Equivalent ratios
Step 2 Share in a ratio (given total, one part or difference)
Step 3 Link ratios and fractions
Step 4 Combine a set of ratios
Step 5 Share in a ratio (algebraically)
Step 6 Solve problems with ratio and algebra
Step 7 Ratios and scales

Keywords

Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios
Integer: whole number, can be positive, negative or zero.
Fraction: represents how many parts of a whole.
Denominator: the number below the line on a fraction. The number represent the total number of parts.
Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Origin: (0,0) on a graph. The point the two axes cross
Gradient: The steepness of a line

Compare with ratio

"For every dog there are 2 cats"

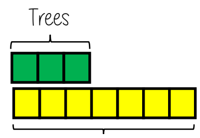
Dogs: Cats
1:2

The ratio has to be written in the same order as the information is given.
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have the be of the same value to compare ratios

Ratios and fraction

Trees: Flowers
3 : 7



Ratio

Fraction of trees

$$\frac{\text{Number of parts of in group}}{\text{Total number of parts}} \quad \frac{3}{10}$$

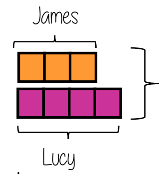
Fraction

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

Whole: £350
7 parts to share between (3 James, 4 Lucy)

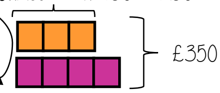
£350 ÷ 7 = £50

□ = one part = £50

Put back into the question

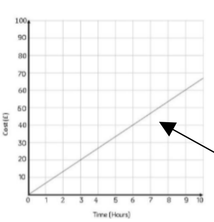
James: Lucy
× 50 3 : 4 × 50
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Ratio and graphs



Graphs with a constant ratio are directly proportional
- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life
× 10 1cm : 30cm × 10
10cm : 300cm

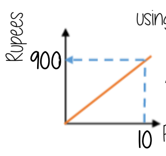
Conversion between currencies

For every £1 I have 90 Rupees

£1 = 90 Rupees ← Currency is directly proportional

× 10 £1 = 90 Rupees × 10
£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

× 7 £1 = 90 Rupees × 7 ← 630 ÷ 90 = 7
£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

The question states that this part has to be 1 unit. Therefore Divide by 4

4 : 20
1 : 5

This side has to be divided by 4 too – to keep in proportion

the n part does not have to be an integer for this type of question

Best buys

4 pens costs £2.60 | 10 pens costs £6.00

"1 pen costs…" £2.60 ÷ 4 = £0.65 | £6.00 ÷ 10 = £0.60

"1-pound buys…" 4 ÷ 2.60 = 1.54 pens | 10 ÷ 6 = 1.67 pens

You could work out how much 40 pens are and then compare

Compare the solution in the context of the question

The best value has the lowest cost "per pen"

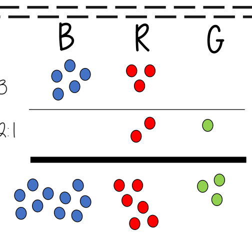
The best value means £1 buys you more pens

Combining ratios

The ratio of Blue counters to Red counters is 5:3

The ratio of Red counters to Green counters is 2:1

Ratio of Blue to Red to Green



10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

YEAR 10H — AUTUMN

N17 - WORK WITH FRACTIONS

Sparx Maths

Add and subtract fractions – U736 Multiply and divide fractions – U475, U544
Solve problems with fractions – U881, U916 Add and subtract algebraic fractions – U685
Multiply algebraic fractions – U457 Divide algebraic fractions – U824
Simplify algebraic fractions – U103, U437, U294
+ or - more complex algebraic fractions (E) – U685 Multiply and divide more complex algebraic fractions (E) – U457, U824 Solve equations with algebraic fractions (E) – U505

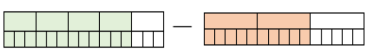
What do I need to be able to do?

Step 1 Add and subtract fractions
Step 2 Multiply and divide fractions
Step 3 Solve problems with fractions
Step 4 Add and subtract algebraic fractions
Step 5 Multiply algebraic fractions
Step 6 Divide algebraic fractions
Step 7 Simplify algebraic fractions
Step 8 Add and subtract more complex algebraic fractions (E)
Step 9 Multiply and divide more complex algebraic fractions (E)
Step 10 Solve equations with algebraic fractions (E)

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Denominator: the number below the line on a fraction. The number represent the total number of parts
Equivalent: of equal value
Mixed numbers: a number with an integer and a proper fraction
Improper fractions: a fraction with a bigger numerator than denominator
Substitute: replace a variable with a numerical value
Place value: The Value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

Add and subtract algebraic fractions

$\frac{4}{5} - \frac{2}{3}$  $= \frac{2}{15}$

$\frac{12}{15} - \frac{10}{15}$

Use equivalent fractions to find a common multiple for both denominators

$$\frac{2}{a} + \frac{1}{b} \quad (\times b, \times a) \quad \frac{2b}{ab} + \frac{1a}{ab}$$

This cant simplify any further… we could combine into one fraction though to get: $\frac{2b + 1a}{ab}$

Algebraic fractions use the same rules as basic fractions, We can only add or subtract things that are the same size (in the same **denomination**).

$$\frac{3x}{4} + \frac{2x}{3} \quad (\times 3, \times 4)$$
$$\frac{9x}{12} + \frac{8x}{12}$$
$$= \frac{9x + 8x}{12}$$
$$= \frac{17x}{12}$$

$$\frac{3}{x+1} + \frac{1}{2} \quad (\times (2), \times (x+1))$$
$$\frac{(3)(2)}{(x+1)(2)} + \frac{(1)(x+1)}{(2)(x+1)}$$
$$\frac{6}{2x+2} + \frac{x+1}{2x+2}$$
$$= \frac{6 + (x+1)}{2x+2} = \frac{x+7}{2x+2}$$

You might get questions which are a bit more complicated, it is still the same process though, All you are trying to do is to get the denominators the same…. think cross multiply if you're struggling

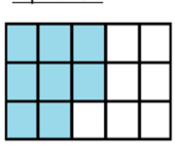
$$\frac{x+1}{x+2} + \frac{x+3}{x+4}$$
$$\frac{(x+1)(x+4)}{(x+2)(x+4)} + \frac{(x+3)(x+2)}{(x+4)(x+2)}$$
$$\frac{x^2+5x+4}{x^2+6x+8} + \frac{x^2+5x+6}{x^2+6x+8}$$
$$\frac{2x^2+10x+10}{x^2+6x+8}$$

Dividing any fractions *Remember to use reciprocals*

$$\frac{2}{5} \div \frac{3}{4}$$
$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

Multiplying any fractions

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Multiply numerators together
and
Multiply denominators together

Simplify:

$$\frac{5x+10}{3} \times \frac{x}{x+2}$$
$$= \frac{(5x+10) \times (x)}{(3) \times (x+2)}$$
$$= \frac{5x^2 + 10x}{3x+6}$$
$$= \frac{(5x)(x+2)}{(3)(x+2)}$$
common factor: $x + 2$
$$= \frac{5x}{3}$$

Simplify:

$$\frac{4}{3x+9} \div \frac{3}{2x+6}$$
$$\frac{4}{3x+9} \times \frac{2x+6}{3}$$
$$= \frac{(4) \times (2x+6)}{(3x+9) \times (3)}$$
$$= \frac{8x+24}{9x+27}$$
$$= \frac{(8)(x+3)}{(9)(x+3)}$$
common factor: $x + 3$
$$= \frac{8}{9}$$

Multiply and divide algebraic fractions

To help, we can **factorise & cancel** *before* multiplying.

$$\frac{7a - 21}{a} \times \frac{3}{4a - 12} = \frac{7 \times (a-3) \times 3}{a \times 4 \times (a-3)}$$
$$= \frac{7 \times 3}{a \times 4} = \frac{21}{4a}$$

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 3 (SPRING 1) :

N18 - NON-CALCULATOR METHODS

A17 - STRAIGHT LINE GRAPHS

P3 – PROBABILITY

N19 - ROUNDING AND ESTIMATING

YEAR 10H — SPRING

N18 - NON-CALCULATOR METHODS

Sparx Maths

Order of operations – M521
Related calculations – M521, M135
Solve multi-step problems – U325
Convert recurring decimals to fractions – U689
Convert more complex recurring decimals to fractions (E) – U689

What do I need to be able to do?

- **Step 1** Order of operations
- **Step 2** Related calculations
- **Step 3** Solve multi-step problems
- **Step 4** Convert recurring decimals to fractions
- **Step 5** Convert more complex recurring decimals to fractions (E)

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
Round: making a number simpler, but keeping its place value close the what it originally was
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value
Integer: Whole numbers; can be positive or negative
Rational Numbers: All integers plus fractions (they are made by diving 2 integers
Irrational numbers: Numbers that cannot be written as a fractions e.g. π and $\sqrt{2}$

Division methods

$3584 \div 7 = 512$

Short division

$$\begin{array}{r|l} & 5\ \ 1\ \ 2 \\ 7 & 3\ {}^{3}5\ \ 8\ {}^{1}4 \end{array}$$

Complex division

$\div 24 = \div 6 \div 4$

Break up the divisor using factors

Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

$24 \div 0.02 \rightarrow 24 \div 0.2 \rightarrow 240 \div 2$

All give the same solution as represent the same proportion.
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods

	H	T	O
	1	8	7
×			9

Long multiplication (column)

×	100	80	7
9			

Grid method

	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
	1	8	7
+	1	8	7

Repeated addition

Start with the representation of 2

Less effective method especially for bigger multiplication

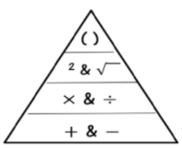
Multiplication with decimals

Perform multiplications as integers
e.g. $0.2 \times 0.3 \rightarrow 2 \times 3$

Make **adjustments** to your answer to match the question: $0.2 \times 10 = 2$
$0.3 \times 10 = 3$

Therefore $6 \div 100 = \underline{0.06}$

Use order or operations



Brackets
Indices or roots
Multiplication or division
Addition or subtraction

Brackets around negative substitutions helps remove calculation errors

×	−3	−2	−1	0	1	2	3
−3	9	6	3	0	−3	−6	−9
−2	6	4	2	0	−2	−4	−6
−1	3	2	1	0	−1	−2	−3
0	0	0	0	0	0	0	0
1	−3	−2	−1	0	1	2	3
2	−6	−4	−2	0	2	4	6
3	−9	−6	−3	0	3	6	9

Remember square roots have a positive and negative value

$$-1\times[(3-4\times7)\div5]-2\times24\div6$$
$$=-1\times[(3-28)\div5]-2\times24\div6$$
$$=-1\times[(-25)\div5]-2\times24\div6$$
$$=-1\times[-5]-2\times24\div6$$
$$=5-2\times24\div6$$
$$=5-48\div6$$
$$=5-48\div6$$
$$=5-8$$
$$=-3$$ ✓

Remember to work down, writing the answer directly underneath the section you've just calculated

Rational and irrational numbers

convert recurring decimals

Integer Whole numbers; can be positive or negative

Recurring decimals are not irrational so you need to be able to convert these to fractions. Using this technique to shown below:

Write $0.\dot{4}\dot{3}$ as a fraction in its simplest form.

$$\begin{aligned} \text{Let} \quad x &= 0.\dot{4}\dot{3} \\ 100x &= 43.\dot{4}\dot{3} \\ 99x &= 43 \\ x &= \frac{43}{99} \end{aligned}$$

$100x - 99x = x$

Rational Numbers All integers plus fractions (they are made by diving 2 integers

Write $0.5\dot{2}\dot{3}$ as a fraction in its simplest form.

$$\begin{aligned} \text{Let} \quad x &= 0.5\dot{2}\dot{3} \\ 10x &= 5.\dot{2}\dot{3} \\ 1000x &= 523.\dot{2}\dot{3} \\ 990x &= 518 \\ x &= \frac{518}{990} = \frac{259}{495} \end{aligned}$$

The goal is to make the recurring parts match

When we get two equations where the recurring decimal parts match, we can subtract them (523-5=518)

Irrational numbers Numbers that cannot be written as a fractions e.g. π and $\sqrt{2}$

Tips

Use **10x**, **100x**, **1000x**, etc., depending on how many digits recur.
Always **simplify your fraction** at the end.
Always check your answer using a calculator if you have one!

YEAR 10H — SPRING

A17 - STRAIGHT LINE GRAPHS 1 OR 2

Sparx Maths

Plot straight line graphs – U741 y = mx + c – U669
Find the equation of a line from a graph – U315
Represent solutions to single inequalities on a graph – U747 Find the midpoint of a line segment – U789
Equation of a straight-line graph given one point and a gradient – U477
Equation of a straight-line graph given two points (E) – U848
Equations of perpendicular lines (E) – U898 Real-life straight-line graphs – U652, U862

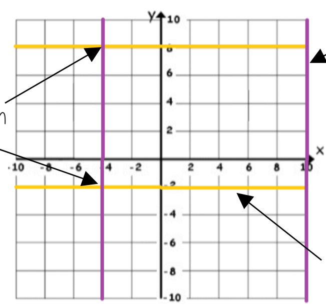
What do I need to be able to do?

Step 1 Plot straight line graphs
Step 2 y = mx + c
Step 3 Find the equation of a line from a graph
Step 4 Represent solutions to single inequalities on a graph
Step 5 Represent solutions to multiple inequalities on a graph
Step 6 Find the midpoint of a line segment
Step 7 Equation of a straight-line graph given one point and a gradient
Step 8 Equation of a straight-line graph given two points (E)
Step 9 Equations of perpendicular lines (E)
Step 10 Real-life straight-line graphs

Keywords

Gradient: the steepness of a line
Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.
Parallel: two lines that never meet with the same gradient.
Co-ordinate: a set of values that show an exact position on a graph.
Linear: linear graphs (straight line) – linear common difference by addition/ subtraction
Asymptote: a straight line that a graph will never meet.
Reciprocal: a pair of numbers that multiply together to give 1.
Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes



Intersection points

All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

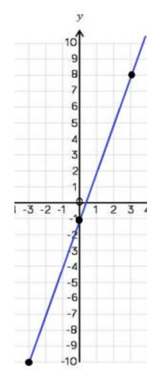
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 × the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

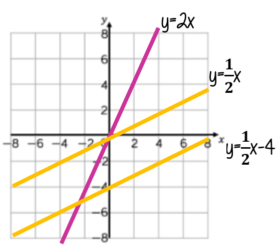
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient

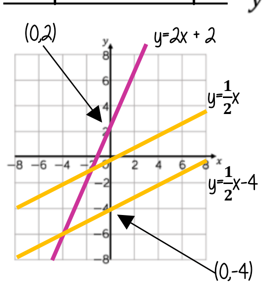
Positive gradients

Negative gradients

Compare Intercepts

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

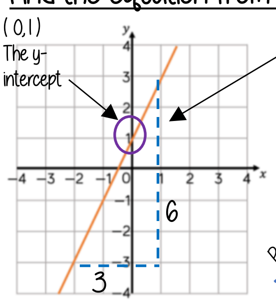
The equation of a line can be rearranged: Eg

y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

<u>Direct Proportion graphs</u> To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

Solve graphically (R)

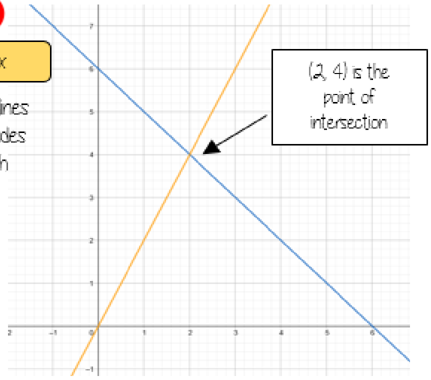
$x + y = 6$

$y = 2x$

Linear equations are straight lines. The point of intersection provides the x and y solution for both equations.

The solution that satisfies both equations is

$x = 2$ and $y = 4$



Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

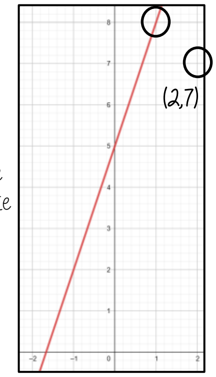
$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5



Equation from two points

Work out the equation of the line that passes through the points $(3, 5)$ and $(6, 14)$

x_1 y_1 x_2 y_2

Gradient $= \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

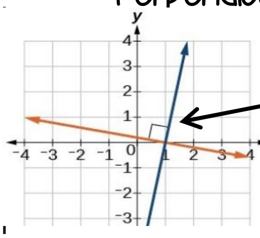
$= \frac{14 - 5}{6 - 3}$

$= \frac{9}{3}$

$m = 3$

Perpendicular lines

Unlike parallel lines, **perpendicular lines** do intersect. Their intersection forms a right or 90-degree angle.



The slope of one line is the negative reciprocal of the slope of the other line. The product of a number and its reciprocal is 1.

Reciprocals

The reciprocal of a number is the number you would have to multiply it by to get the answer 1.

4 The reciprocal is $\frac{1}{4}$

$\frac{2}{3}$ The reciprocal is $\frac{3}{2}$

0.25 $\frac{1}{4}$ The reciprocal is 4

Write the decimal as a fraction first!

The product of the gradients of a pair of perpendicular lines will always be -1 therefore you need to find the negative reciprocal

-3 The negative reciprocal is $\frac{1}{3}$

5 The negative reciprocal is $-\frac{1}{5}$

Equations for Perpendicular lines

Example 1 Line a $y = 2x + 1$

Line B is perpendicular and passes through (2, 4). Find the equation of Line B.

$y = mx + c$

gradient (m) = negative reciprocal of 2

$y = -\frac{1}{2}x + c$

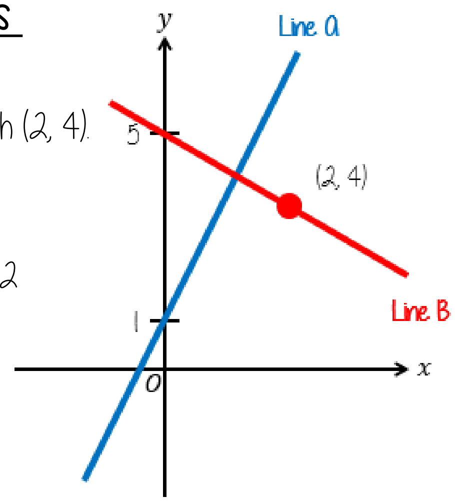
Substitute (2,4) to find c

$4 = -(0.5 \times 2) + c$

$4 = -1 + c$

$5 = c$

$y = -\frac{1}{2}x + 5$ Or $y = 5 - \frac{1}{2}x$



Example 2

Line L_2 is perpendicular to $y = 1 - \frac{1}{3}x$ and passes through (5, 7).

Find the equation of Line L_2.

Gradient is the negative reciprocal of this line

$y = mx + c$ → $y = 3x + c$

substitute x & y which are (5, 7) to find c

$7 = (3 \times 5) + c$

$7 = 15 + c$

$-8 = c$

Line L_2: $y = 3x - 8$

YEAR 10H — SPRING

P3 — PROBABILITY

Sparx Maths

Find the probability of a single event – U510 Use the property that probabilities sum to 1 – U510
List and count outcomes – U104 Relative frequency – U580
Sample spaces for 1 or more events – U104
Two-way tables and frequency trees – M899, U280 Independent events – U558
Tree diagrams for independent events – U558 Tree diagrams for dependent events – U729
Conditional probability (Tree diagrams) (E) – U821

What do I need to be able to do?

Step 1 Find the probability of a single event
Step 2 Use the property that probabilities sum to 1
Step 3 List and count outcomes
Step 4 Relative frequency
Step 5 Sample spaces for 1 or more events
Step 6 Two-way tables and frequency trees
Step 7 Independent events
Step 8 Tree diagrams for independent events
Step 9 Tree diagrams for dependent events
Step 10 Conditional probability (Tree diagrams)

Keywords

Event: one or more outcomes from an experiment
Outcome: the result of an experiment.
Intersection: elements (parts) that are common to both sets
Union: the combination of elements in two sets.
Expected Value: the value/ outcome that a prediction would suggest you will get
Universal Set: the set that has all the elements
Systematic: ordering values or outcomes with a strategy and sequence
Product: the answer when two or more values are multiplied together.

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times

$0.3 \times 100 = 30$

Single event probability

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(white chocolate) = 1 – 0.15 – 0.35
= 0.5

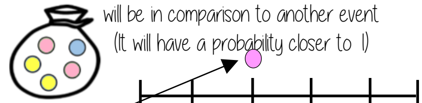
The probability scale

Impossible
0 or 0%

Even chance
0.5, $\frac{1}{2}$ or 50%

Certain
1 or 100%

The more likely an event the further up the probability it will be in comparison to another event
(It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Experimental data

Theoretical probability What we expect to happen

Experimental probability What actually happens when we try it out

The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
Theoretical probability is proportional

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

60 → Adult 26 → Elephant 13, Other 13
60 → Children 34 → Elephant 24, Other 10

Two-way table

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Sample space

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

The possible outcomes from tossing a coin

P (Even number and tails) = $\frac{3}{12}$

Independent events

The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

$P(A \text{ and } B) = P(A) \times P(B)$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick. Because they are replaced the second pick has the same probability

3 out of 5 are blue — Blue $\frac{3}{5}$
2 out of 5 are yellow — Yellow $\frac{2}{5}$

Blue → Blue $\frac{3}{5}$ $= P(BB)$ $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$
Blue → Yellow $\frac{2}{5}$ $= P(BY)$ $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$
Yellow → Blue $\frac{3}{5}$ $= P(YB)$ $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$
Yellow → Yellow $\frac{2}{5}$ $= P(YY)$ $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

Sum of probabilities = 1

Dependent events

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks, Jamie picks 2 socks from the drawer.

Pick first sock: black $\frac{5}{9}$, white $\frac{4}{9}$

Black → black $\frac{4}{8}$ $= P(BB)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$
Black → white $\frac{4}{8}$ $= P(BW)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$
White → black $\frac{5}{8}$ $= P(WB)$ $\frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$
White → white $\frac{3}{8}$ $= P(WW)$ $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

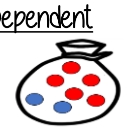
A white sock has been removed

Sum of probabilities = 1

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick

Independent and Dependent

Independent events are events which do not affect one another.



Dependent events affect one another's probabilities.
This is also known as **conditional probability**.

YEAR 10H – SPRING

N19 - ROUNDING AND ESTIMATING

Sparx Maths

Round to decimal places and significant figures – U298, U965
Estimate answers to calculations – M878
Use of a calculator – U161
Error intervals (including truncation) – U657, U301, U108
Upper and lower bounds – U587

What do I need to be able to do?

Step 1 Round to decimal places and significant figures
Step 2 Estimate answers to calculations
Step 3 Use of a calculator
Step 4 Error intervals (including truncation)
Step 5 Upper and lower bounds

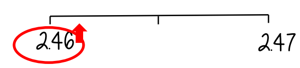
Keywords

Significant figure: The digits in a number that carry meaning contributing to its precision (starting from the first non-zero digit).
Rounding: Reducing the digits in a number while keeping its value close to the original.
Approximation: A value or quantity that is nearly but not exactly correct.
Estimate: A rough calculation of the value, number, or quantity.
Error interval: A range within which a number lies after rounding.
Upper bound: The highest possible value in an error interval.
Lower bound: The lowest possible value in an error interval.
Accuracy: How close a measured or calculated value is to the true value.

Using a Calculator

Check Mode – Make sure it's in **Degrees** and **Maths/Comp** mode.
Brackets – Use them to keep the correct order of operations.
Negatives – Use the (–) key for negative numbers.
Squares/Roots – Use X^2 for squares and $\sqrt{x}$ for roots.
Fractions – Use the fraction button to enter and simplify.
Standard Form – Use the EXP or $x10^x$ button.
ANS – Reuse your last answer with the ANS key.

Rounding



This shows the number is closer to 2.46

Significant Figures
370 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

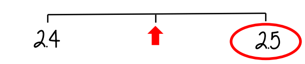
SF: Round to the first nonzero number

Round to decimal places

Focus on the numbers **after** the decimal point

"To 1dp" – to one number after the decimal
"To 2dp" – to two numbers after the decimal

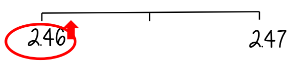
2.46192 (to 1dp) - Is this closer to 2.4 or 2.5



2.4|6192 This shows the number is closer to 2.5

2.46|192 This shows the number is closer to 2.46

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47



Estimate the calculation

Round to 1 significant figure to estimate

$4.2 + 6.7 \approx 4 + 7 \approx 11$

This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

$21.4 \times 3.1 \approx 20 \times 3 \approx 60$

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Upper and lower bounds

When completing calculations involving boundaries we are aiming to find the greatest or smallest answer.

The boundaries of a number derive from **rounding**.
State the boundaries of 360 when it has been rounded to 2 significant figures:
$$355 \leq x < 365$$

State the boundaries of 4.5 when it has been rounded to 2 decimal place:
$$4.45 \leq x < 4.55$$

These boundaries can also be called the **error interval** of a number.

	+	-	×	÷
Upper bound	$UB_1 + UB_2$	$UB_1 - LB_2$	$UB_1 \times UB_2$	$\frac{UB_1}{LB_2}$
Lower bound	$LB_1 + LB_2$	$LB_1 - UB_2$	$LB_1 \times LB_2$	$\frac{LB_1}{UB_2}$

$$D = \frac{x}{y}$$

x = 99.7 correct to 1 decimal place.
y = 67 correct to 2 significant figures.
Work out an upper and lower bounds for

$$\text{Upper bound} = \frac{UB_1}{LB_2}$$

$$\text{Lower bound} = \frac{LB_1}{UB_2}$$

$$99.65 \leq x < 99.75$$

error intervals for x and y

$$66.5 \leq y < 67.5$$

$$\text{Upper bound } D = \frac{99.75}{66.5} = 1.5$$

$$\text{Lower bound } D = \frac{99.65}{67.5} = 1.48$$

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 4 (Spring 2):

G13 - PERIMETER, AREA AND VOLUME

S5 - INTERPRET AND REPRESENT DATA

A18 - NON-LINEAR GRAPHS

YEAR 10H – SPRING

G13 - PERIMETER, AREA AND VOLUME

Sparx Maths

Perimeter of a 2-D shape – U630, M690, M635
Area of a 2-D shape – M900, M231, M390
Area and circumference of a circle – M169, M231, U459
Arc length and perimeter – U221 Area of a sector – U373
Volume of a prism – M722 Volume of a cylinder – U915 Nets – M518
Surface area of a prism – M661 Surface area of a cylinder – U464

What do I need to be able to do?

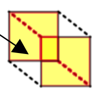
- **Step 1** Perimeter of a 2-D shape
- **Step 2** Area of a 2-D shape
- **Step 3** Area and circumference of a circle
- **Step 4** Arc length and perimeter
- **Step 5** Area of a sector
- **Step 6** Volume of a prism
- **Step 7** Volume of a cylinder
- **Step 8** Nets
- **Step 9** Surface area of a prism
- **Step 10** Surface area of a cylinder

Keywords

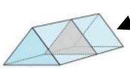
2D: two dimensions to the shape e.g. length and width
3D: three dimensions to the shape e.g. length, width and height
Vertex: a point where two or more lines segments meet
Edge: a line on the boundary joining two vertex
Face: a flat surface on a solid object
Cross-section: a view inside a solid shape made by cutting through it
Plan: a drawing of something when drawn from above (sometimes birds eye view)
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Recognise prisms

A solid object with two identical ends and flat sides



The cross section will also be identical to the end faces.



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

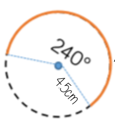
Arc length

Remember an arc is part of the circumference
Circumference of the whole circle $= \pi d = \pi \times 9 = 9\pi$

Arc length $= \frac{\theta}{360} \times$ circumference

$= \frac{240}{360} \times 9\pi$

$= \frac{2}{3} \times 9\pi = 6\pi$



Perimeter

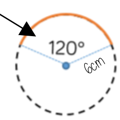
Perimeter is the length around the outside of the shape
This includes the arc length and the radii that encloses the shape
$= 6\pi + 9$

Perimeter $= \frac{\theta}{360} \times$ circumference $+ 2r$

Sector area

Remember a sector is part of a circle

Area of the whole circle $= \pi r^2 = \pi \times 6^2 = 36\pi$

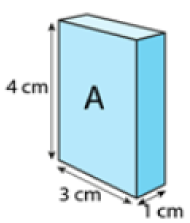


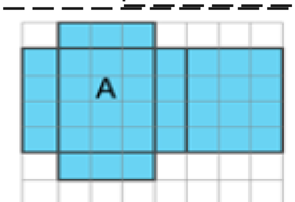
$= \frac{120}{360} \times 36\pi$

$= \frac{1}{3} \times 36\pi = 12\pi$

Sector area $= \frac{\theta}{360} \times$ area of circle

Nets of cuboids

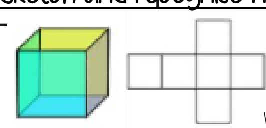


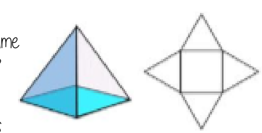


1cm grids help to draw accurately

Visualise the folding of the net
Will it make the cuboid with all sides touching

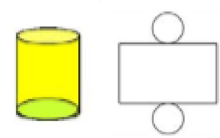
Sketch and recognise nets

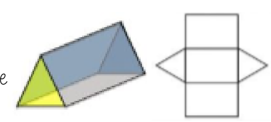




Do they have the same number of faces?

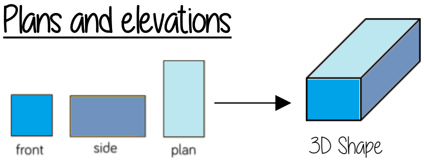
Where do the edges join?





Are the shapes of the faces correct?

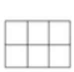
Plans and elevations



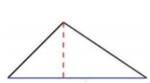
The direction you are considering the shape from determines the front and side views

Area of 2D shapes

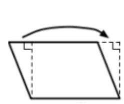
Rectangle
Base x Height



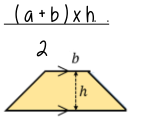
Triangle
½ x Base x Perpendicular height



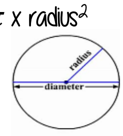
Parallelogram/ Rhombus
Base x Perpendicular height



Area of a trapezium
$\frac{(a+b) \times h}{2}$

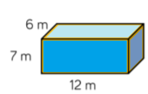


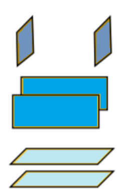
Area of a circle
π x radius²



Surface area

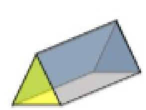
Sketching nets first helps you visualise all the sides that will form the overall surface area





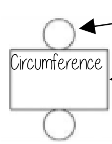
For cubes and cuboids you can also find one of each face and double it

Sides	6 x 7	
	6 x 7	
Front and back	12 x 7	
	12 x 7	
Top and Bottom	12 x 6	
	12 x 6	Sum of all sides is surface area



For other shapes – not all the sides are the same, so calculate the individually

Surface area - cylinders



The area of the circle
π x radius²

The width of this face is the same as the circumference
π x diameter x height

2 x π x radius² + π x diameter x height

Volumes

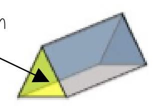
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative

Cross section



Cross section

Prisms and cylinders = area cross section x height

Height can also be described as depth

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 10H — SPRING

S5 - INTERPRET AND REPRESENT DATA 1 OF 2

Sparx Maths

Averages and range – M328, M934, M841 Averages from an ungrouped frequency table – M899, M127, U312 Mean from a grouped frequency table – M940
Averages from a grouped frequency table – M127, M440, U854
Use data to compare distributions – U520, U879, U837 Types of data – U322 Sampling – U162
Capture and recapture – U328 Scatter graphs – U199, U277

What do I need to be able to do?

Step 1 Averages and range
Step 2 Averages from an ungrouped frequency table
Step 3 Mean from a grouped frequency table
Step 4 Averages from a grouped frequency table
Step 5 Use data to compare distributions
Step 6 Types of data
Step 7 Sampling
Step 8 Capture and recapture
Step 9 Scatter graphs
Step 10 Interpolation and extrapolation

Keywords

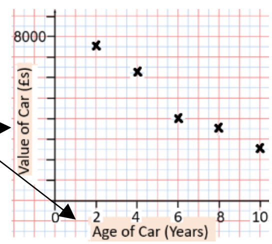
Data: Information collected for analysis; can be qualitative (words) or quantitative (numbers).
Outlier: A value that is much higher or lower than the rest of the data.
Error: A mistake in data collection or recording.
Mean: The average, found by adding all values and dividing by the number of values.
Median: The middle value when data is in order.
Mode: The most frequent value in a data set.
Range: The difference between the highest and lowest values.
Frequency Table: A table showing how often each value or group of values occurs.
Grouped Data: Data that is organized into intervals or classes.
Distribution: The way data is spread out, often compared using averages and range.

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled

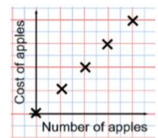


"This scatter graph show as the age of a car increases the value decreases"

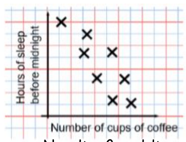
The link between the data can be explained verbally

The axis should fit all the values on and be equally spread out

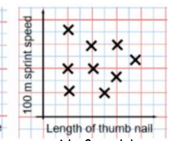
Linear Correlation



Positive Correlation



Negative Correlation

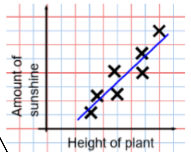


No Correlation

Positive Correlation	Negative Correlation	No Correlation
As one variable increases so does the other variable	As one variable increases the other variable decreases	There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph



Things to know:
- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

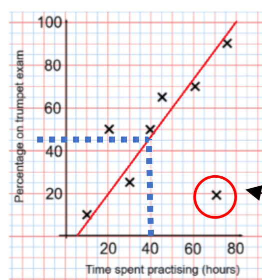
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data
This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "outlier"
It is an outlier because it doesn't fit this model and stands apart from the ~~data~~

Averages from lists

The Mean

A measure of average to find the central tendency… a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

55 ÷ 5 | Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (It does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order

4, 8, 8, 11, 24

Find the value in the middle

4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data The modal group – which group has the highest frequency

Averages from a table

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \leq 50$	1	45	45
$50 < x \leq 60$	3	65	195
$60 < x \leq 70$	5	65	325

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

YEAR 10H — SPRING

S5 - INTERPRET AND REPRESENT DATA 2 OF 2

Sparx Maths

Averages and range – M328, M934, M841 Averages from an ungrouped frequency table – M899, M127, U312 Mean from a grouped frequency table – M940
Averages from a grouped frequency table – M127, M440, U854
Use data to compare distributions – U520, U879, U837 Types of data – U322 Sampling – U162
Capture and recapture – U328 Scatter graphs – U199, U277

What do I need to be able to do?

- Step 1 Averages and range
- Step 2 Averages from an ungrouped frequency table
- Step 3 Mean from a grouped frequency table
- Step 4 Averages from a grouped frequency table
- Step 5 Use data to compare distributions
- Step 6 Types of data
- Step 7 Sampling
- Step 8 Capture and recapture
- Step 9 Scatter graphs
- Step 10 Interpolation and extrapolation

Keywords

Data: Information collected for analysis; can be qualitative (words) or quantitative (numbers).
Outlier: A value that is much higher or lower than the rest of the data.
Error: A mistake in data collection or recording.
Mean: The average, found by adding all values and dividing by the number of values.
Median: The middle value when data is in order.
Mode: The most frequent value in a data set.
Range: The difference between the highest and lowest values.
Frequency Table : A table showing how often each value or group of values occurs.
Grouped Data : Data that is organized into intervals or classes.
Distribution: The way data is spread out, often compared using averages and range.

Construct a stratified sample

260 people work in a school.
175 of these are teachers, and the rest do other jobs.

A survey is going to be given to 55 of the staff.

Work out the number of teachers and the number of other staff members there should be in the sample.

Teachers: $\frac{175}{260} \times 55 = 37.019\ldots \approx 37$

$55 - 37 = 18$

37 teachers.
18 other staff members.

How might the sample be split into other strata?

Male and females or Staff with and without student contact etc.

Compare distributions

When comparing distributions, you look at one of the averages and measure of spread; at Foundation level this will always be the range at higher this may be the interquartile range. The average is used as an indicator of overall performance and the range is used to describe the consistency.

Compare the finish times of both groups of runners.

1st group

Median (minutes)	64
IQR (minutes)	26

On average, the 2nd group were faster, as the median finish time for the 1st group (64 minutes), was higher than the median finish time of the 2nd group (61 minutes).

2nd group

Median (minutes)	61
IQR (minutes)	17.5

The finish times of the 2nd group were more consistent, as the IQR of the finish times of the 2nd group (17.5 minutes) was lower than the IQR of the finish times of the 1st group (26 minutes).

Capture and recapture

Capture-recapture is a method used to estimate the size of a population when it's difficult or impossible to count every individual directly.

Here's a more detailed breakdown:

1. **Capture and Mark:** A sample of the population is captured and marked (e.g., tagged, dyed, or banded) in a way that doesn't harm them.
2. **Release:** The marked individuals are released back into the population.
3. **Recapture:** After a suitable period for mixing, a second sample is captured.
4. **Count:** The number of marked individuals in the second sample is counted.
5. **Estimate:** The population size is estimated using a formula, based on the proportion of marked individuals in the second sample.

Formula:
A common formula used for estimating the population size (N) is:
N = (M * C) / R

Where:
M = Number of individuals initially marked
C = Total number of individuals captured in the second sample
R = Number of marked individuals recaptured in the second sample

Sophie is trying to work out the total number of fish in a lake.
One day she captures 30 fish, marks them, and then returns them.
On the next day, she captures 40 fish and finds that 8 of them are marked. Work out an estimate for the total number of fish in the lake.

n is the unknown population.

What fraction of the whole population were marked? $\frac{30}{n}$

What fraction of the second sample were marked? $\frac{8}{40}$

Set these equal and solve for an estimate to n.

$$\frac{30}{n} = \frac{8}{40}$$

$$1200 = 8n$$

$$150 = n$$

Assumptions:

- The population remains relatively stable between the two capture events (no significant births, deaths, immigration, or emigration).
- The marked individuals mix randomly with the rest of the population.
- The marking method doesn't affect the survival or behaviour of the marked individuals.
- All individuals in the population have an equal chance of being captured in both samples.

YEAR 10H – SPRING

A18 - NON-LINEAR GRAPHS

Sparx Maths

Quadratic graphs – U989 Intercepts and roots of quadratic graphs – U601
Turning points – U769 Cubic graphs – U980
Approximate solutions to equations using graphs – U601
Equation of the tangent to a curve – U800
Estimate the area under a curve (E) – U882 Equation of a circle – U567
Equation of a tangent to a circle (E) – U567

What do I need to be able to do?

- **Step 1** Quadratic graphs
- **Step 2** Intercepts and roots of quadratic graphs
- **Step 3** Turning points
- **Step 4** Cubic graphs
- **Step 5** Approximate solutions to equations using graphs
- **Step 6** Equation of the tangent to a curve
- **Step 7** Estimate the area under a curve (E)
- **Step 8** Equation of a circle
- **Step 9** Equation of a tangent to a circle (E)

Keywords

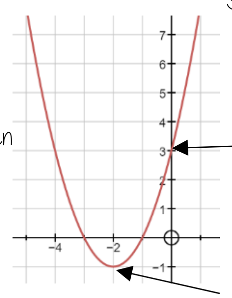
Parabola: a 'u' shaped curve that has mirror symmetry.
Quadratic: a curved graph with the highest power being 2. Square power.
Root: the x-value where the graph crosses the x-axis (solution).
Intercept: where the graph meets the x-axis or y-axis.
Turning point: The highest or lowest point on a quadratic graph.
Cubic: a graph with the highest power being 3. W-shaped or S-shaped curve.
Inflection point: where a cubic graph changes curvature.
Tangent: a line that touches a curve at exactly one point.
Gradient: the steepness or slope of a line or curve.
Estimate: use a graph to find an approximate solution or area.

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

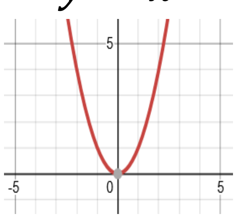
Coordinate pairs for plotting $(-3, 0)$

Plot all of the coordinate pairs and join the points with a curve (freehand)

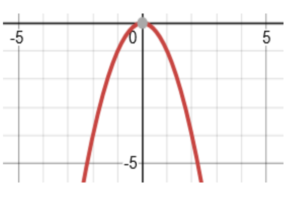
Quadratic graphs are always symmetrical with the turning point in the middle

a quadratic graph will always be in the shape of a parabola.

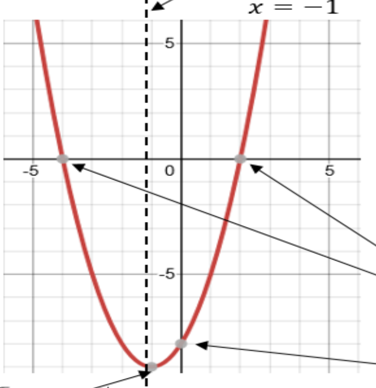
$y = x^2$



$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.



Line of symmetry $x = -1$

Examples
$y = x^2 + 2x - 8$

a quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation.
This is dependant on how many times the graph crosses the x axis.

Roots $x = -4$
$x = 2$

y intercept = -8

Turning point $(-1, -9)$

Interpreting graphs

Interpret other graphs

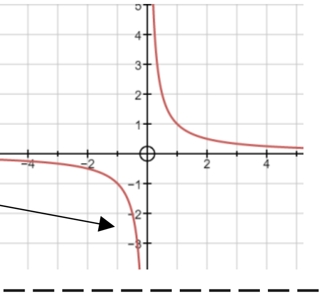
Cubic Graphs

$y = x^3 + 2x^2 - 2x + 1$

If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

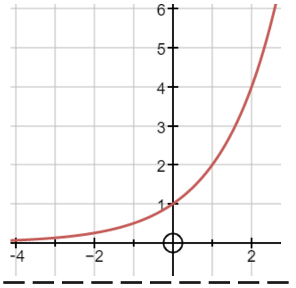


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x



area under a curve

Here is the graph $y = -x^3 + 3x + 4$ By drawing suitable trapezia, estimate the area under the curve between

$x = -2$ and $x = 1$

Area $= \frac{1}{2} \times (6 + 2) \times 1$

$= 4$ units²

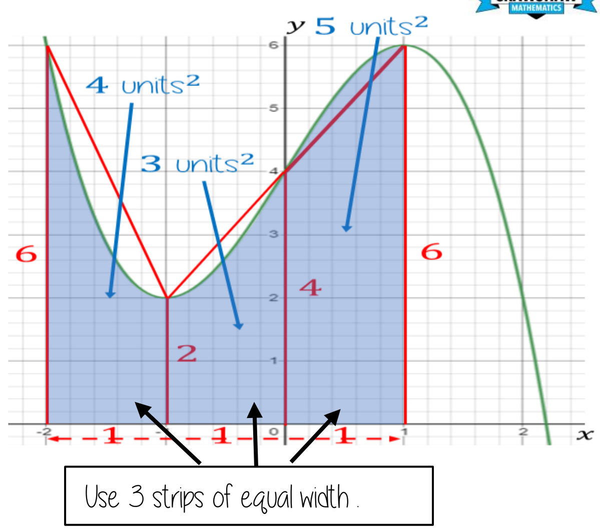
Area $= \frac{1}{2} \times (2 + 4) \times 1$

$= 3$ units²

Area $= \frac{1}{2} \times (4 + 6) \times 1$

$= 5$ units²

Total area under curve ≈ 12 units²

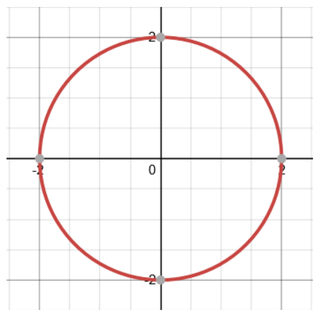


Equations of a Circle

The **equation of a circle** will be in the format:

$$x^2 + y^2 = radius^2$$

The **centre** of each circle will be at the coordinate **(0,0)**.



$x^2 + y^2 = 4$

$Radius = \sqrt{4}$

$= \pm 2$

$gradient = \frac{1}{2}$

Therefore we can plot the following coordinates to support us sketching our graph: (0,2), (0,-2), (2,0), (-2,0)

Tangent to a Circle

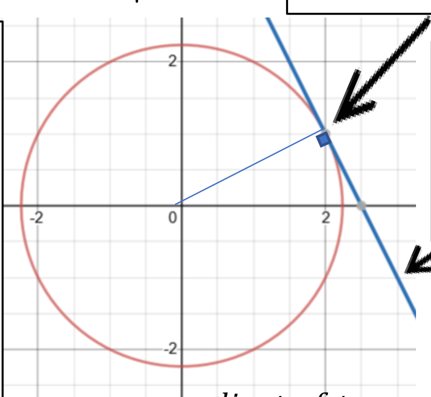
A **tangent** touches a circle at **one point**.

A **tangent** line is **perpendicular** to the **radius** of the circle.

Find the equation of the tangent to the circle with equation:

$$x^2 + y^2 = 5$$

which passes through the point (2, 1)



The gradient of the tangent is the **negative reciprocal** of the gradient of the equation of the line of the radius.

Step 1: Find the equation of the line which is the radius of the circle.

$therefore\ y = \frac{1}{2}x$

Step 2: The tangent is perpendicular to the radius.

$gradient\ of\ tangent = negative\ reciprocal\ of\ \frac{1}{2}$ $= -2$

$y = -2x + c$

Step 3: Substitute in the given coordinate (2, 1) in to $y = -2x + c$ to find c

When x = 2 and y = 1 from the coordinate (2, 1)

$1 = (-2 \times 2) + c$

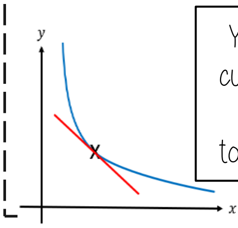
$1 + 4 = c$

$5 = c$

Therefore, the Equation of the tangent is:

$$y = -2x + 5$$

Tangent to a Curves



You can also get tangents on curves. Be careful though, make sure your tangent is only touching the curve at one point

The tangent to the curve at $(3, 1)$ has been drawn. Now find the equation of the tangent. $y = mx + c$

$m = 1$ $\quad x = 3, y = 1$

$1 = 1 \times 3 + c$

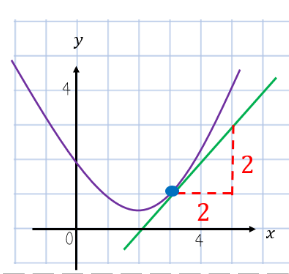
$1 = 3 + c$

$c = 1 - 3$

$c = -2$

Equation of the tangent is

$y = x - 2$



Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 5 (summer 1):

G14 - ANGLES

S6 - GRAPHS AND DIAGRAMS

G15 - VECTORS

YEAR 10H — SUMMER

G14 - ANGLES

Angles around a point, on a straight line and vertically opposite – U390, U655, U730
Angles in a triangle – U628 Angles in a quadrilateral – U732
Exterior angles of a polygon – U427 Interior angles of a polygon – U427
Solve problems with angles in polygons (E) – U427, U887 Alternate and corresponding angles – U826
Alternate, corresponding and co-interior angles – U826
Solve problems with angles and algebra – U655, U325, U870 Prove geometric facts (E) – U471

What do I need to be able to do?

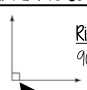
- **Step 1** Angles around a point, on a straight line and vertically opposite
- **Step 2** Angles in triangles and quadrilaterals
- **Step 3** Exterior angles of any polygon
- **Step 4** Interior angles of any polygon
- **Step 5** Solve problems with angles in polygons
- **Step 6** Alternate, corresponding and co-interior angles
- **Step 7** Solve problems with angles in parallel lines
- **Step 8** Solve problems with angles and algebra
- **Step 9** Prove geometric facts (E)

Keywords

Parallel: Straight lines that never meet
Angle: The figure formed by two straight lines meeting (measured in degrees)
Transversal: A line that cuts across two or more other (normally parallel) lines
Isosceles: Two equal size lines and equal size angles (in a triangle or trapezium)
Polygon: A 2D shape made with straight lines
Sum: Addition (total of all the interior angles added together)
Regular polygon: All the sides have equal length; all the interior angles have equal size.

Basic angle rules and notation

Acute Angles
0°< angle <90°

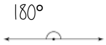


Right Angles
90°

Right angle notation

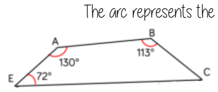
Obtuse
90°< angle <180°

Reflex
180°< angle <360°



Straight Line
180°

The letter in the middle is the angle
The arc represents the part of the angle



Angle Notation: three letters **ABC**
This is the angle at B = 113 °
Line Notation: two letters **EC**
The line that joins E to C

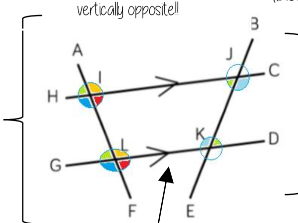
Vertically opposite angles
Equal
Angles around a point
360 °

Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!!

Lines AF and BE are **transversals** (lines that bisect the parallel lines)

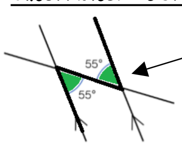
Corresponding angles often identified by their **"F shape"** in position



Alternate angles often identified by their **"Z shape"** in position

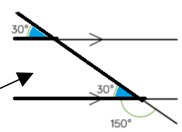
This notation identifies parallel lines

Alternate/ Corresponding angles

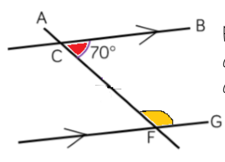


Because alternate angles are equal the highlighted angles are the same size

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

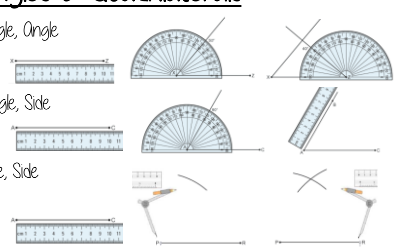


Because co-interior angles have a sum of 180° the highlighted angle is 110°

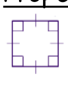
As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Triangles & Quadrilaterals

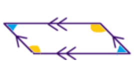
Side, Angle, Angle
Side, Angle, Side
Side, Side, Side



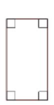
Properties of Quadrilaterals



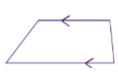
Square
All sides equal size
All angles 90°
Opposite sides are parallel



Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles



Rectangle
All angles 90°
Opposite sides are parallel



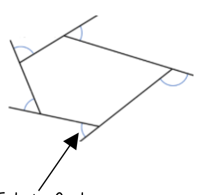
Trapezium
One pair of parallel lines

Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Rhombus
All sides equal size
Opposite angles are equal

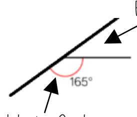
Sum of exterior angles

Exterior angles all add up to 360°



Exterior Angles
Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles



Exterior Angle
Interior Angle

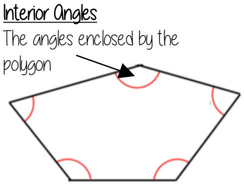
Interior angle + Exterior angle = straight line = 180°
Exterior angle = 180 – 165 = **15°**

Number of sides = 360° ÷ exterior angle
Number of sides = 360 ÷ 15 = **24 sides**

Sum of interior angles

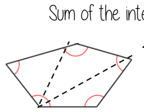
(number of sides – 2) x 180

Interior Angles
The angles enclosed by the polygon



This is an **irregular** polygon – the sides and angles are different sizes

Sum of the interior angles = (5 – 2) x 180

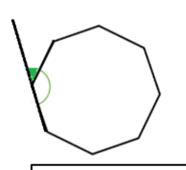


This shape can be made from three triangles
Each triangle has 180°

Sum of the interior angles = 3 x 180
= 540°

Remember this is **all** of the interior angles added together

Missing angles in regular polygons



Exterior angle = 360 ÷ 8 = 45°

Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8}$ = 135°

Exterior angles in regular polygons = 360° ÷ number of sides

Interior angles in regular polygons = $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

YEAR 10H – SUMMER

S6 - GRAPHS AND DIAGRAMS

1 OF 2

Sparx Maths

Pie charts – M574, M165, U508 Time-series graphs – M932, M544, M140
Frequency polygons – U840 Stem-and-leaf diagrams – M648, M210, U200
Draw histograms – U185, U814 Interpret histograms – U983
Draw cumulative frequency diagrams – U182
Interpret cumulative frequency diagrams – U642

What do I need to be able to do?

Step 1 Pie charts
Step 2 Time-series graphs
Step 3 Frequency polygons
Step 4 Stem-and-leaf diagrams
Step 5 Draw histograms
Step 6 Interpret histograms
Step 7 Draw cumulative frequency diagrams
Step 8 Interpret cumulative frequency diagrams

Keywords

Pie Chart: A circular chart divided into sectors, each showing a part of the whole in percentages.
Sector: A slice of a pie chart representing a part of the total
Time-Series Graph: Shows how data changes over time at regular intervals.
Axis: The horizontal or vertical lines used to plot data on graphs.
Frequency: How often a value or category appears in a dataset.
Frequency Polygon: A line graph connecting midpoints of class intervals to show data shape.
Stem-and-Leaf Diagram: Organizes numbers by splitting them into "stems" (leading digits) and "leaves" (last digits)
Histogram: A bar graph where bars touch, used to show grouped continuous data.
Class Interval: A group or range of values used in histograms or frequency tables.
Cumulative Frequency: A running total of frequencies, used to find medians and quartiles.
Cumulative Frequency Diagram: A graph used to estimate medians, quartiles, and spread

Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4

<u>Stem and leaf diagrams:</u>
Must include a key to explain what it represents
The information in the diagram should be ordered

<u>Back to back stem and leaf diagrams</u>

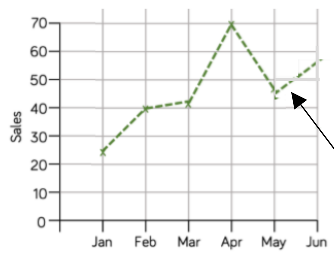
Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

15 | 3
Means 153 cm tall

<u>Back to back stem and leaf diagrams</u>
Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

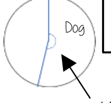
Draw and interpret Pie Charts

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs



<u>Multiple method</u>
As 60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

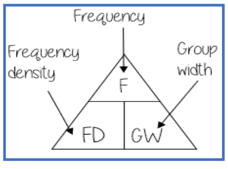
Use a protractor to draw
This is 192°

$\frac{32}{60} \times 360 = 192°$

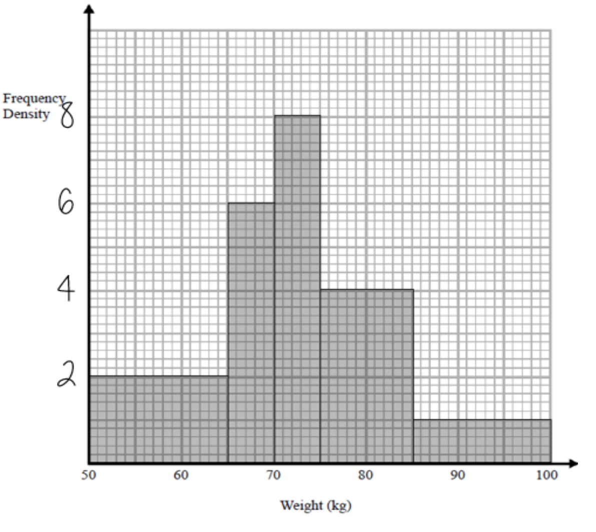
<u>Comparing Pie Charts:</u>
You NEED the overall frequency to make any comparisons

Histograms

A **Histogram** is a graphical representation of data consisting of rectangles whose **area** is **proportional to the frequency** of a variable and whose **width** is **equal to the group width**.



Weight	Frequency	Frequency density
50 < w ≤ 65	30	30 / 15 = 2
65 < w ≤ 70	30	30 / 5 = 6
70 < w ≤ 75	40	40 / 5 = 8
75 < w ≤ 85	40	40 / 10 = 4
85 < w ≤ 100	15	15 / 15 = 1



YEAR 10H — SUMMER

S6 - GRAPHS AND DIAGRAMS

2 OF 2

Sparx Maths

Pie charts – M574, M165, U508 Time-series graphs – M932, M544, M140
Frequency polygons – U840 Stem-and-leaf diagrams – M648, M210, U200
Draw histograms – U185, U814 Interpret histograms – U983
Draw cumulative frequency diagrams – U182
Interpret cumulative frequency diagrams – U642

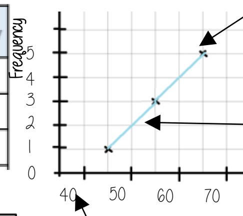
What do I need to be able to do?

- Step 1 Pie charts
- Step 2 Time-series graphs
- Step 3 Frequency polygons
- Step 4 Stem-and-leaf diagrams
- Step 5 Draw histograms
- Step 6 Interpret histograms
- Step 7 Draw cumulative frequency diagrams
- Step 8 Interpret cumulative frequency diagrams

Frequency polygon

We do not know from grouped data where each value is placed so have to use an estimate for calculations

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5



Each point is plotted at them mid point for the group it represents

Each point is connected with a straight line

The data about weight starts at 40. So the axis can start at 40

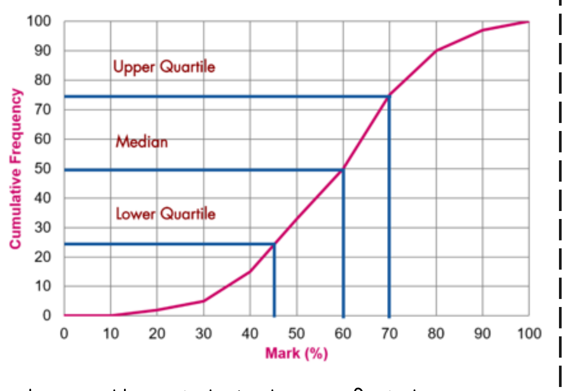
MID POINTS
Mid-points are used as estimated values for grouped data. The middle of each group

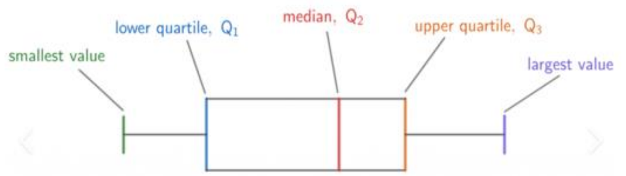
Mid-point
$\frac{\text{Start point + End point}}{2}$

Core concepts

A **cumulative frequency** graph shows a running total of frequency.

We can read the **median** and the **interquartile range** from this graph.





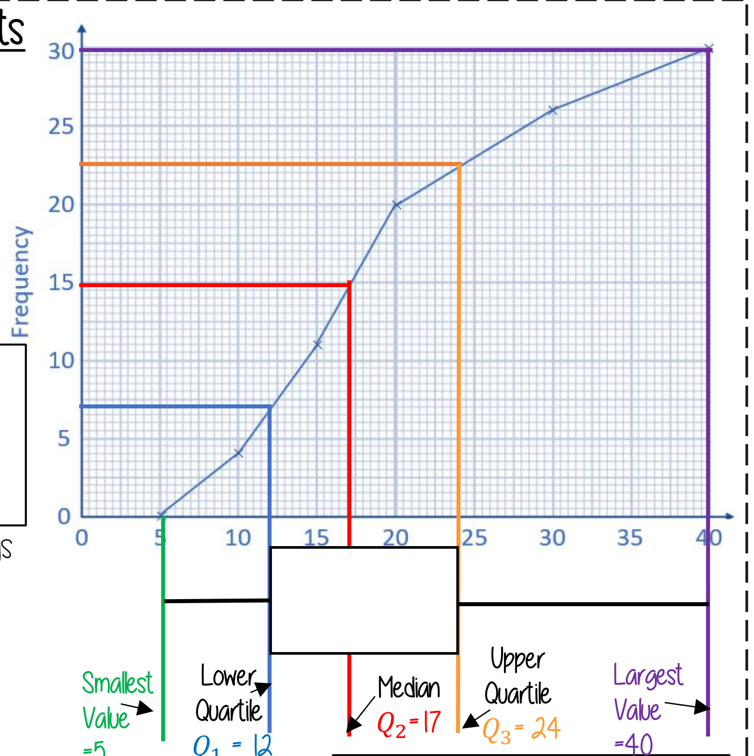
A **box plot** shows the distribution of data using **minimum, maximum, median** and **quartiles**.

cumulative frequency diagrams and box plots

The table shows the heights of 30 plants.

Height (h)	Frequency	CF
$5 \leq h < 10$	4	4
$10 \leq h < 15$	7	11
$15 \leq h < 20$	9	20
$20 \leq h < 30$	6	26
$30 \leq h < 40$	4	30

Add a column for CF (the running total)



When plotting a cumulative frequency diagram we always plot on the end of the range since it is a running total

Median and quartiles are found from the y axis:
Lower quartile = 25% of the way through the data
Median = 50% of the way through the data
Upper quartile = 75% of the way through the data
Interquartile range = UQ – LQ

Interquartile Range = $Q_3 - Q_1$
12= 24-12

Range = Largest value – smallest value
35= 40-5

Sparx Maths

Understand and represent vectors – U632 Vector notation – U632
Vectors multiplied by a scalar – U564 Add vectors – U903
Add and subtract vectors – U903 Vector journeys in shapes – U781
Vectors in quadrilaterals – U781 Parallel vectors – U660

What do I need to be able to do?

- **Step 1** Understand and represent vectors
- **Step 2** Vector notation
- **Step 3** Vectors multiplied by a scalar
- **Step 4** Add vectors
- **Step 5** Add and subtract vectors
- **Step 6** Vector journeys in shapes
- **Step 7** Vectors in quadrilaterals
- **Step 8** Parallel vectors

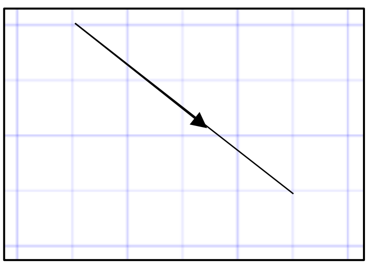
Keywords

Direction: the line our course something is going
Magnitude: the magnitude of a vector is its length
Scalar: a single number used to represent the multiplier when working with vectors
Column vector: a matrix of one column describing the movement from a point
Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet

Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another

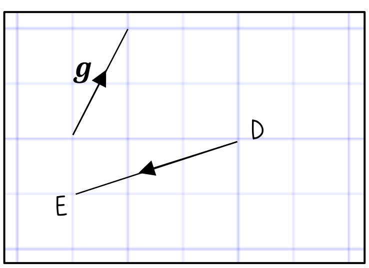
Movement along the x-axis → $\binom{4}{-3}$ ← Movement along the y-axis



Vectors show both direction and magnitude

The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

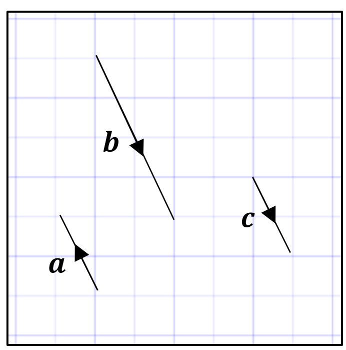
$\overrightarrow{DE} = \binom{-3}{-1}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \binom{1}{2}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

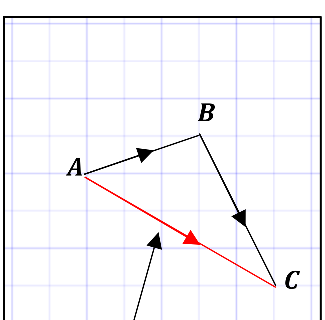
The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{a} = \binom{-1}{2}$ $\boldsymbol{b} = \binom{2}{-4}$ $\boldsymbol{c} = \binom{1}{-2}$

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

Addition of vectors

$\overrightarrow{AB} = \binom{3}{1}$ $\overrightarrow{BC} = \binom{2}{-4}$



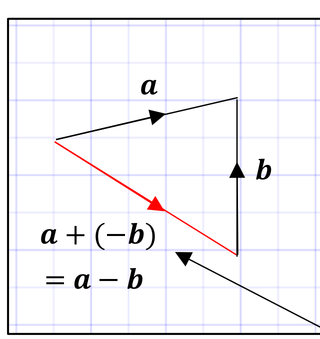
The resultant

$$\overrightarrow{AB} + \overrightarrow{BC}$$
$$= \binom{3}{1} + \binom{2}{-4}$$
$$= \binom{3+2}{1+-4}$$
$$\overrightarrow{AC} = \binom{5}{-3}$$

Look how this addition compares to the vector $\overrightarrow{AC}$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \binom{5}{-3}$$

Addition and subtraction of vectors



$\boldsymbol{a} = \binom{5}{1}$ $\boldsymbol{b} = \binom{0}{4}$

$\boldsymbol{a} + (-\boldsymbol{b}) = \binom{5 + \;\; -0}{1 + \;\; -4} = \binom{5}{-4}$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1

YEAR 10H – SUMMER

G15 – VECTORS

2 OF 2

Sparx Maths

Understand and represent vectors – U632 Vector notation – U632
Vectors multiplied by a scalar – U564 Add vectors – U903
Add and subtract vectors – U903 Vector journeys in shapes – U781
Vectors in quadrilaterals – U781 Parallel vectors – U660

What do I need to be able to do?

Step 1 Understand and represent vectors
Step 2 Vector notation
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Step 8 Parallel vectors

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Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet

Key Concepts

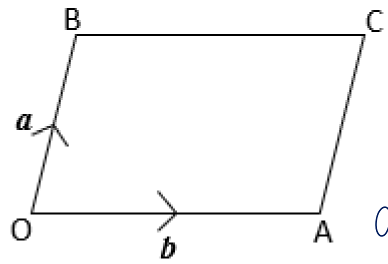
Vectors notation: $\boldsymbol{a}$ $\overrightarrow{AB}$ $\underline{a}$

Magnitude: Length of the arrow
Direction: Where the arrow is pointing
Travelling against an arrow **changes the sign** of the vector

Parallel lines of **equal length** have the **same** vector.
Parallel lines of **different lengths** have a **multiple** of the vector.

For two vectors to form a **straight line** they must have vector values which are **multiples of one another** and must have a **common point**.

Vectors in shapes



OA = $\boldsymbol{b}$ OB = $\boldsymbol{a}$
OABC is a parallelogram.
M is the midpoint of AC.

a) State the vector of OC.

As BC is parallel and equal in length to OA, it has the vector value of $\boldsymbol{b}$.
Therefore OC $= \boldsymbol{a} + \boldsymbol{b}$

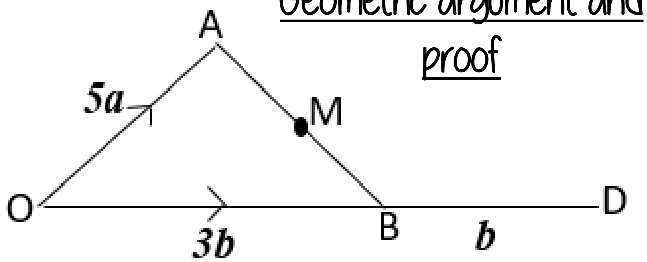
b) State the vector of AO.

As we are travelling against the arrow, the vector changes sign.
Therefore AO = $-\boldsymbol{b}$

c) State the vector of OM.

As AC is parallel and equal in length to OB, is has the vector value of $\boldsymbol{a}$.
M is the midpoint of AC.
Therefore OM $= \boldsymbol{b} + \frac{1}{2}\boldsymbol{a}$

Geometric argument and proof



C is the point such that OC:CA = 4:1
M is the midpoint of AB.
D is the point such that OB:OD = 3:4
Show that C, M and D are on the same straight line.

$\overrightarrow{CA} = \frac{1}{5}\overrightarrow{OA}$
$= \frac{1}{5}(5a)$
$= a$

$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM}$
$= a + \frac{1}{2}(-5a + 3b)$
$= a - 2.5a + 1.5b$
$= -1.5a + 1.5b$

$\overrightarrow{MD} = \overrightarrow{MB} + \overrightarrow{BD}$
$= \frac{1}{2}(-5a + 3b) + 4b$
$= -2.5a + 1.5b + b$
$= -2.5a + 2.5b$

C, M and D are on a **straight line** as CM and MD are *multiples* of one another and have the **common point** of M.

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 6 (Summer 2):

N20 - FACTORS AND POWERS

G16 - PYTHAGORAS' THEOREM AND TRIGONOMETRY

A19 - SIMULTANEOUS EQUATIONS

YEAR 10H – SUMMER

N20 - FACTORS, POWERS AND SURDS 1 OF 3

Sparx Maths

Prime factorisation, HCF and LCM – U250 Powers, roots and negative indices – U235, U694
Fractional indices – U985, U772 Four operations with surds – U633, U872
Simplify surds – U338 Expand single brackets with surds – U499
Rationalise the denominator – U707 Expand double brackets with surds – U499
Rationalise the denominator with more complex denominators (E) – U281
Solve problems with surds – U633, U872

What do I need to be able to do?

- Step 1 Prime factorisation, HCF and LCM
- Step 2 Powers, roots and negative indices
- Step 3 Fractional indices
- Step 4 Four operations with surds
- Step 5 Simplify surds
- Step 6 Expand single brackets with surds
- Step 7 Rationalise the denominator
- Step 8 Expand double brackets with surds
- Step 9 Rationalise the denominator with more complex denominators (E)
- Step 10 Solve problems with surds

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Coefficient: The number used to multiply a variable

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

This list continues and doesn't end

$3x$, $6x$, $9x$ …

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

45 is not a multiple of 3 because it is 3 x 15 ← Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10: 1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

| x | x | x | x | x | x |

$6x \times 1$ OR $6 \times x$

Factors of $6x$: 6, x, 1, $6x$, $2x$, 3, $3x$, 2

$2x \times 3$

$3x \times 2$

Prime numbers

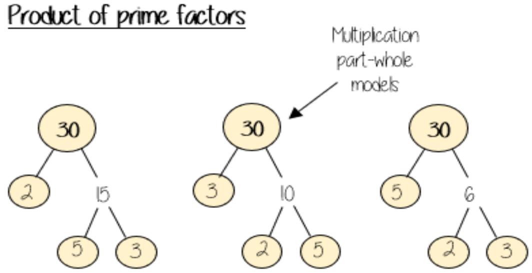
- Integer
- Only has 2 factors
- and itself

2 – The first prime number. The only even prime number

Learn or how-to quick recall…

2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

Product of prime factors



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg 60	30 x 2	2 x 3 x 5 **x 2**
150	30 x 5	2 x 3 x 5 **x 5**

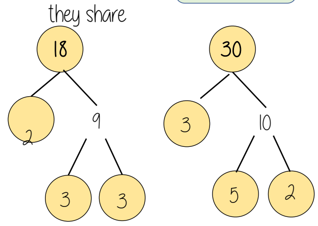
Finding the HCF and LCM

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share — HCF = 6

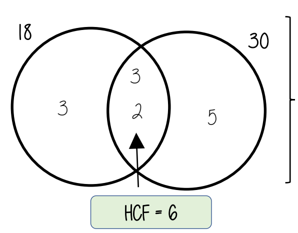


LCM – Lowest common multiple

LCM of 18 and 30

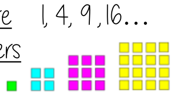
18	18, 36, 54, 72, 90
30	30, 60, 90

The first time their multiples match — LCM = 90

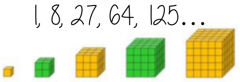


Square and cube numbers

Square numbers 1, 4, 9, 16…



Cube numbers 1, 8, 27, 64, 125…



Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$

$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$

Looking at the sequen can help to understan negative powers

FRACTIONAL INDICES

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

Square root: $25^{\frac{1}{2}} = \sqrt{25} = 5$

Cube root: $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$25^{\frac{3}{2}} = (\sqrt[2]{25})^3 = 5^3 = 125$

Remember that this is the same as $(25^{\frac{1}{2}})^3$

NEGATIVE FRACTIONAL INDICES

$$a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$$

$25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}} = \frac{1}{5^3} = \frac{1}{125}$

Remember this is the same as $(25^{\frac{1}{2}})^3$

$8^{-\frac{1}{3}} = \frac{1}{8^{\frac{1}{3}}} = \frac{1}{2}$

Remember this means the cube root of 8!

YEAR 10H — SUMMER

N20 - FACTORS, POWERS AND SURDS 2 OF 3

Sparx Maths

Prime factorisation, HCF and LCM – U250 Powers, roots and negative indices – U235, U694
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What do I need to be able to do?

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Understand and use surds

A **surd** is a square root which cannot be reduced to a whole number. They are irrational numbers, which when written in decimal form, would go on forever.

Examples and Non-Examples

Examples: $\sqrt{5}$, $5\sqrt{6}$, $\sqrt{2}$, $3\sqrt{2}$, $\sqrt{3}$, $\sqrt{11}$, $\sqrt{197}$

Non-Examples:
- $\sqrt{4}$ — this can be simplified to 2, which is a rational number
- $\sqrt[3]{27}$ — this can be simplified to 3, which is a rational number
- $(\sqrt{5})^2$ — this can be simplified to 5, which is a rational number

Simplifying Surds

Method 1

Here we are looking for the largest square number which is also a factor of 24

Simplify $\sqrt{24}$

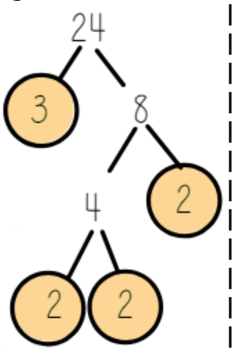
Factors of 24:
1 x 24
2 x 12
3 x 8
4 x 6

So $\sqrt{24} = \sqrt{4 \times 6}$
$= \sqrt{4} \times \sqrt{6}$
$= 2\sqrt{6}$

Method 2

Simplify $\sqrt{24}$

Using prime factor decomposition and our knowledge that $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, we can say:



$24 = 2 \times 2 \times 2 \times 3$
So $\sqrt{24} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{3}$
$= 2 \times \sqrt{2} \times \sqrt{3}$
$= 2\sqrt{6}$

Formulas to Remember

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

$\sqrt{a} \times \sqrt{a} = a$

$\sqrt{a} + \sqrt{a} = 2\sqrt{a}$

$2\sqrt{a} - \sqrt{a} = \sqrt{a}$

Adding & Subtracting Surds

coefficients are dealt with just like they are in algebra!

$4\sqrt{3} + 7\sqrt{3} = 11\sqrt{3}$

$8\sqrt{2} - 5\sqrt{2} = 3\sqrt{2}$

$\sqrt{5} + \sqrt{5} = 2\sqrt{5}$ ← think of this like x + x, or 2 lots of x

$\sqrt{3}$ and $\sqrt{5}$ are **UNLIKE TERMS** so this $2\sqrt{3} - 7\sqrt{5}$ cannot be simplified any further

It is important to try and simplify your surds before working with them so you don't miss things like this →

$\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$

$\sqrt{12} = \sqrt{4} \times \sqrt{3}$
$= 2 \times \sqrt{3}$
$= 2\sqrt{3}$

$\sqrt{27} = \sqrt{9} \times \sqrt{3}$
$= 3 \times \sqrt{3}$
$= 3\sqrt{3}$

YEAR 10H – SUMMER

N20 - FACTORS, POWERS AND SURDS 3 OF 3

Sparx Maths

Prime factorisation, HCF and LCM – U250 Powers, roots and negative indices – U235, U694
Fractional indices – U985, U772 Four operations with surds – U633, U872
Simplify surds – U338 Expand single brackets with surds – U499
Rationalise the denominator – U707 Expand double brackets with surds – U499
Rationalise the denominator with more complex denominators (E) – U281
Solve problems with surds – U633, U872

What do I need to be able to do?

Step 1 Prime factorisation, HCF and LCM
Step 2 Powers, roots and negative indices
Step 3 Fractional indices
Step 4 Four operations with surds
Step 5 Simplify surds
Step 6 Expand single brackets with surds
Step 7 Rationalise the denominator
Step 8 Expand double brackets with surds
Step 9 Rationalise the denominator with more complex denominators (E)
Step 10 Solve problems with surds

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero.
Coefficient: The number used to multiply a variable

Multiplying and dividing Surds

Simplify:

$4\sqrt{20} \times 2\sqrt{3} = 8\sqrt{20 \times 3}$
$= 8\sqrt{60}$
$= 8\sqrt{4}\sqrt{15}$
$= 16\sqrt{15}$

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

$3\sqrt{40} \div \sqrt{2} = 3\sqrt{40 \div 2}$
$= 3\sqrt{20}$
$= 3\sqrt{4}\sqrt{5}$
$= 6\sqrt{5}$

coefficients are dealt with just like they are in algebra!

Expanding brackets with surds

Expand and simplify $\sqrt{3}(2 + \sqrt{6})$

X	2	+ $\sqrt{5}$
$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{18}$

$= 2\sqrt{3} + \sqrt{18}$
$= 2\sqrt{3} + 3\sqrt{2}$

Always remember to check if you can simplify your surds

We can treat this just like we do double brackets in algebra!

Expand and simplify $(1 + \sqrt{3})(\sqrt{2} - 1)$

X	1	+ $\sqrt{3}$
$\sqrt{2}$	$\sqrt{2}$	$\sqrt{6}$
-1	-1	$-\sqrt{3}$

Rationalising Denominators

Rationalise the denominator and simplify $\frac{1}{\sqrt{6}}$

Rationalising the denominator means we are making the denominator of the fraction a RATIONAL number (e.g, not a surd!).

We don't want to CHANGE the value of the fraction but we need to find an equivalent fraction with a rational denominator
We do this by multiplying by '1', in this case;

$$\frac{1}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

Rationalise the denominator and simplify $\frac{2}{3 + \sqrt{2}}$

Remember (x+ y)(x - y)= x - y ?
This result is very important here!
We are left with only two square numbers, and we know that means no surds!

We call (x-y) the conjugate of (x+y), the conjugate of $3 + \sqrt{2}$ is $3 - \sqrt{2}$.

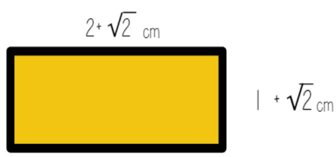
$$\frac{2}{3 + \sqrt{2}} \times \frac{3 - \sqrt{2}}{3 - \sqrt{2}}$$

Special kind of '1'

$$= \frac{2(3 - \sqrt{2})}{(3 + \sqrt{2})(3 - \sqrt{2})} = \frac{6 - 2\sqrt{2}}{9 - 2} = \frac{6 - 2\sqrt{2}}{7}$$

Problem solving with surds

Calculate the area and perimeter of this rectangle, leaving your answer in exact form.



Perimeter:
$1 + \sqrt{2} + 2 + \sqrt{2} + 1 + \sqrt{2} + 2 + \sqrt{2}$
$= 6 + 4\sqrt{2}$ cm

Exact form means we do not round our answer.

Area:
$(1 + \sqrt{2})(2 + \sqrt{2})$
$= 2 + \sqrt{2} + 2\sqrt{2} + 2$
$= 4 + 3\sqrt{2}\ cm^2$

This is why surds are so useful as they are an exact value!

YEAR 10H – SUMMER

G16 - PYTHAGORAS' THEOREM AND TRIGONOMETRY 1 OF 2

Sparx Maths

Pythagoras' theorem (find the hypotenuse) – U385 Pythagoras' theorem (find any side) – U828
Identify hypotenuse, opposite and adjacent sides – U283 Ratios in right-angled triangles – U605
Use trigonometric ratios to find a side– U283 Use trigonometric ratios to find an angle – U545
Exact trigonometric values (E) – U627 Trigonometry in 3D shapes – U170
Area of a non-right-angled triangle (sine area rule) – U592
Use the sine rule – U952 Use the cosine rule– U591

What do I need to be able to do?

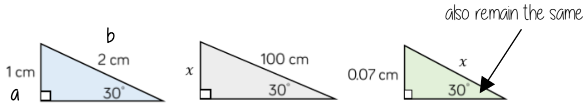
Step 1 Pythagoras' theorem (find any side)
Step 2 Use trigonometric ratios to find an unknown side length
Step 3 Use trigonometric ratios to find an unknown angle
Step 4 Exact trigonometrical values
Step 5 Trigonometry in 3-D shapes
Step 6 Area of a non-right-angled triangle
Step 7 Use the sine rule to find an unknown length
Step 8 Use the sine rule to find an unknown angle
Step 9 Use the cosine rule to find an unknown length
Step 10 Use the cosine rule to find an unknown angle

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Constant: a value that remains the same
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.
Inverse: function that has the opposite effect.
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

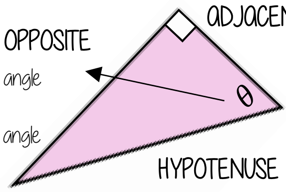
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

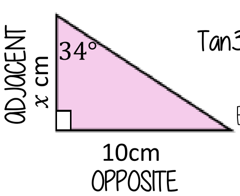
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



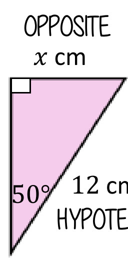
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

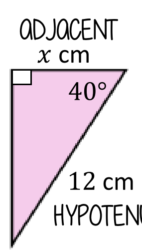
$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The $\text{Sin}(x)$ ratio is the same as the $\text{Cos}(90-x)$ ratio

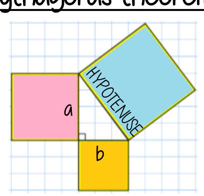


$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem



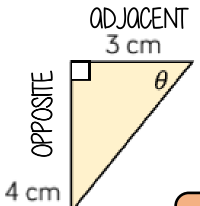
$\text{Hypotenuse}^2 = a^2 + b^2$

This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\text{Tan}\theta = \frac{3}{4}$

$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

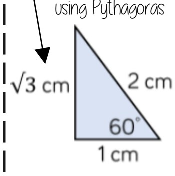
$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles

This side could be calculated using Pythagoras

Because trig ratios remain the same for similar shapes you can generalise from the following statements.



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

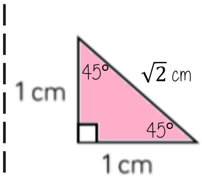
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$

$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$

$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

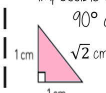
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



$\text{Sin}0 = 0$

$\text{Sin}90 = 1$

$\text{Cos}0 = 1$

$\text{Cos}90 = 0$

YEAR 10H – SUMMER
G16 - PYTHAGORAS' THEOREM AND TRIGONOMETRY 2 OF 2

Sparx Maths
Pythagoras' theorem (find the hypotenuse) – U385 Pythagoras' theorem (find any side) – U828
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What do I need to be able to do?

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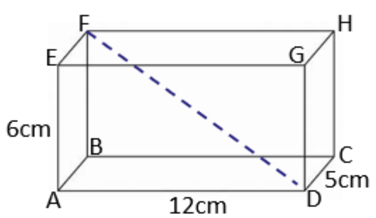
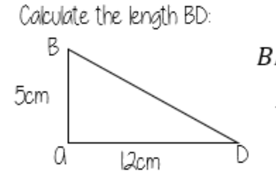
Trig in 3D

The **plane** of a cuboid is a flat 2 dimensional surface. An example of a plane is ABCD.

An example of a **diagonal** in a cuboid is FD.



Calculate the length BD:

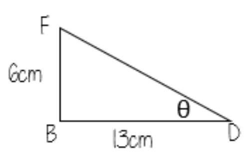


$BD^2 = 12^2 + 5^2$

$BD = \sqrt{169}$

$BD = 13cm$

Calculate the angle between FD and the plane ABCD:

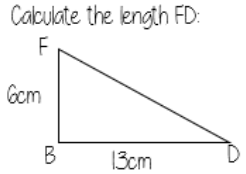


$tan\theta = \frac{6}{13}$

$\theta = tan^{-1}\left(\frac{6}{13}\right)$

$\theta = 24.78^o$

Calculate the length FD:



$FD^2 = 13^2 + 6^2$

$FD = \sqrt{205}$

$FD = 14.32cm$

H

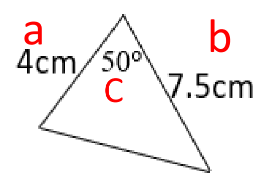
Area of a triangle

H

Area of a triangle using sine

$$area = \frac{1}{2}absinC$$

Remember: Capital letters are angles and sides are little letters



$area = \frac{1}{2} \times 4 \times 7.5 \times sin50$

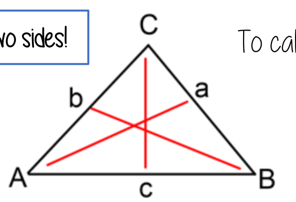
$area = 11.49cm^2$

Sine Rule

Two Angles and Two sides!

To calculate a missing side:

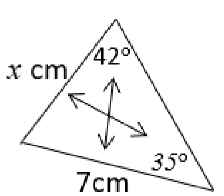
$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$



To calculate a missing angle:

H

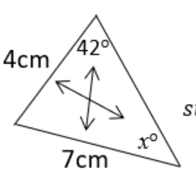
$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$



$\frac{x}{sin35} = \frac{7}{sin42}$

$x = \frac{sin35 \times 7}{sin42}$

$x = 6.0cm$



$\frac{sinx}{4} = \frac{sin42}{7}$

$sinx = \frac{sin42 \times 4}{7}$

$x = sin^{-1}\left(\frac{sin42 \times 4}{7}\right)$

$x = 22.5^o$

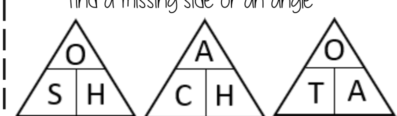
Pythagoras' theorem and basic trigonometry both work with **right angled triangles**.

Pythagoras' Theorem – used to find a missing length when two sides are known

$a^2 + b^2 = c^2$

c is always the hypotenuse (the longest side)

Basic trigonometry SOHCAHTOA – used to find a missing side or an angle



When finding the missing angle we must press **SHIFT** on our calculators first.

Cosine rule

One Angles and Three sides!

To calculate a missing side:

$$a^2 = b^2 + c^2 - 2bccosA$$

To calculate a missing angle:

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

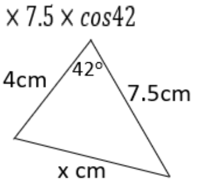
H

$a^2 = b^2 + c^2 - 2bccosA$

$x^2 = 4^2 + 7.5^2 - 2 \times 4 \times 7.5 \times cos42$

$x^2 = 27.66$

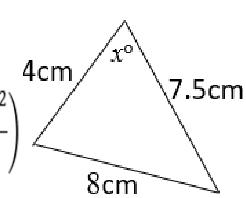
$x = \sqrt{27.66} = 5.26cm$



$cosA = \frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}$

$A = cos^{-1}\left(\frac{4^2 + 7.5^2 - 8^2}{2 \times 4 \times 7.5}\right)$

$A = 82.1^0$



YEAR 10H — SUMMER

A19 - SIMULTANEOUS EQUATIONS 1 OF 2

Sparx Maths

Solve simultaneous equations using graphs – U836 Solve simultaneous equations (adjust both) – U760
Solve simultaneous equations by substitution – U757 Solve problems with simultaneous equations – U137
Solve simultaneous equations (one linear, one non-linear) using graphs (E) – U875
Solve simultaneous equations (one linear, one non-linear) by equating expressions (E) – U547
Solve simultaneous equations (one linear, one non-linear) using substitution (E) – U547

What do I need to be able to do?

Step 1 Solve simultaneous equations using graphs
Step 2 Solve simultaneous equations (no adjustments)
Step 3 Solve simultaneous equations (adjust one)
Step 4 Solve simultaneous equations (adjust both)
Step 5 Solve simultaneous equations by substitution
Step 6 Solve problems with simultaneous equations
Step 7 Solve simultaneous equations (one linear, one non-linear) using graphs (E)
Step 8 Solve simultaneous equations (one linear, one non-linear) by equating expressions (E)
Step 9 Solve simultaneous equations (one linear, one non-linear) using substitution (E)

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position.
Intersection: the point two lines cross or meet.

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

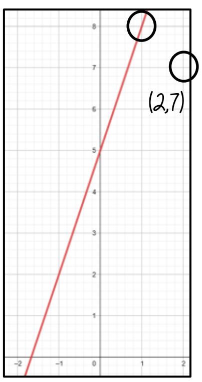
Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5



Is (2,7) on the same line?

$7 \neq 3(2) + 5$

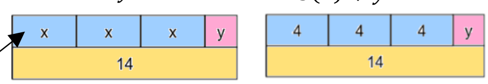
No 7 does NOT equal 6+5

Substituting known variables

A line has the equation 3x + y = 14

Stephanie knows the point x = 4 lies on that line. Find the value for y

$x = 4$



$3x + y = 14$

$3(4) + y = 14$

Two different variables, two solutions

$12 + y = 14$

$-12 \quad -12$

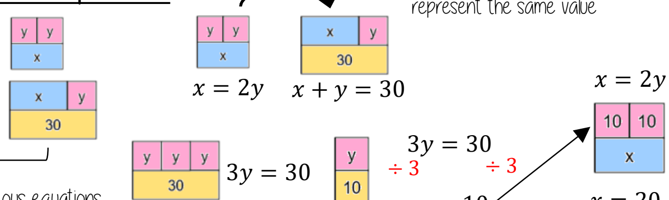
$y = 2$

Substituting in an expression

Substitute 2y in place of the x variable as they represent the same value

x = 2y

x + y = 30



Pair of simultaneous equations (two representations)

$x = 2y \quad x + y = 30$

$3y = 30$

$3y = 30$

$\div 3 \quad \div 3$

$y = 10$

$x = 2y$

$x = 20$

Solve graphically

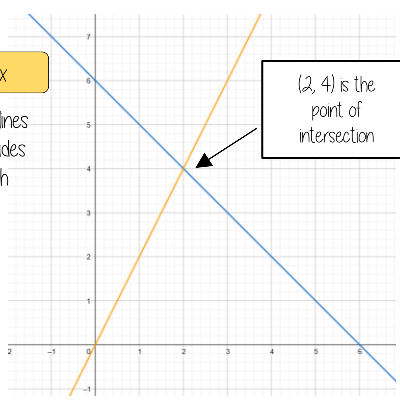
x + y = 6

y = 2x

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations

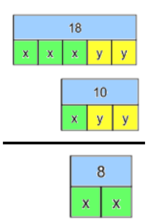
The solution that satisfies both equations is

$x = 2$ and $y = 4$



(2, 4) is the point of intersection

Solve by subtraction



$3x + 2y = 18$

$-\quad x + 2y = 10$

$2x = 8$

$\div 2 \quad \div 2$

$x = 4$

$x + 2y = 10$

$(4) + 2y = 10$

$-4 \quad -4$

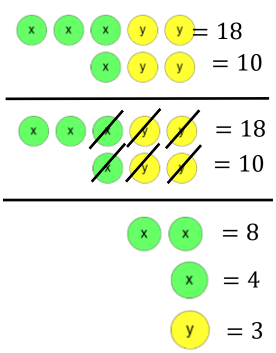
$2y = 6$

$\div 2 \quad \div 2$

$y = 3$

$x = 4$

$y = 3$



Solve by addition

Addition makes zero pairs

$3x + 2y = 16$

$+\ 6x - 2y = 2$

$9x = 18$

$\div 9 \quad \div 9$

$x = 2$

$3x + 2y = 16$

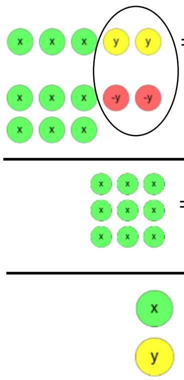
$3(2) + 2(y) = 16$

$6 + 2y = 16$

$-6 \quad -6$

$2y = 10$

$y = 5$



Solve by adjusting one

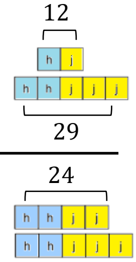
$h + j = 12$

$2h + 2j = 29$

No equivalent values

$2h + 2j = 24$

$2h + 2j = 29$

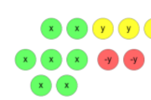


By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$

$5x - 2y = -7$

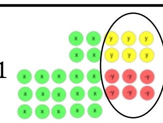


Use LCM to make equivalent x OR y values.
Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$

$15x - 6y = -21$

Now solve by addition



Addition makes zero pairs

A19 - SIMULTANEOUS EQUATIONS 2 OF 2

Sparx Maths

Solve simultaneous equations using graphs – U836 Solve simultaneous equations (adjust both) – U760
Solve simultaneous equations by substitution – U757 Solve problems with simultaneous equations – U137
Solve simultaneous equations (one linear, one non-linear) using graphs (E) – U875
Solve simultaneous equations (one linear, one non-linear) by equating expressions (E) – U547
Solve simultaneous equations (one linear, one non-linear) using substitution (E) – U547

What do I need to be able to do?

Step 1 Solve simultaneous equations using graphs
Step 2 Solve simultaneous equations (no adjustments)
Step 3 Solve simultaneous equations (adjust one)
Step 4 Solve simultaneous equations (adjust both)
Step 5 Solve simultaneous equations by substitution
Step 6 Solve problems with simultaneous equations
Step 7 Solve simultaneous equations (one linear, one non-linear) using graphs (E)
Step 8 Solve simultaneous equations (one linear, one non-linear) by equating expressions (E)
Step 9 Solve simultaneous equations (one linear, one non-linear) using substitution (E)

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position.
Intersection: the point two lines cross or meet.

Solving quadratic simultaneous equations

When we have a **quadratic** and a **linear** equation, we cannot solve by elimination. Instead, we must use the **substitution** method.

Solve:

$$y = x + 1$$
$$y = x^2 + 3x - 2$$

We know what y is so substitute y into the other equation

$x + 1 = x^2 + 3x - 2$

$0 = x^2 + 2x - 3$

Rearrange and solve to find X

$0 = (x + 3)(x - 1)$

$x = -3$ or $x = 1$

Substitute the value of X into an original equation to find y

$y = -3 + 1$
$= -2$

$y = 1 + 1$
$= 2$

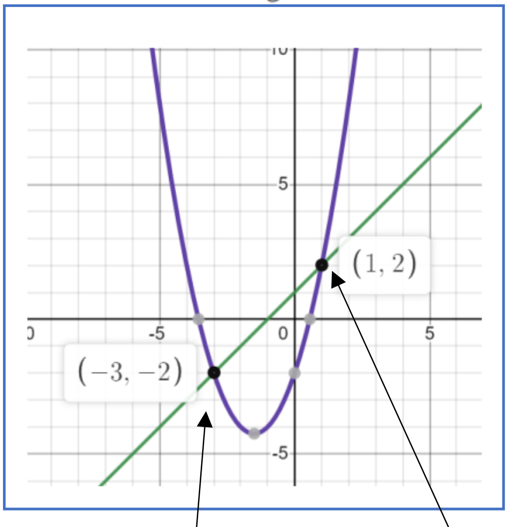
$x = -3$, $y = -2$

$x = 1$, $y = 2$

Solving quadratic simultaneous equations graphically

When could solve the **quadratic** and a **linear** equation graphically to check our solutions:

If you plot the graph of $y = x + 1$ and $y = x^2 + 3x - 2$



where they intercept (cross) is the solution, you will have two values for X and two values for Y

Solving simultaneous equations with a third unknown

Find x and y in terms of k

$$8x - y = k \quad (1)$$
$$2x + 3y = 10k \quad (2)$$

Take (1) and rearrange to find y

$$8x = y + k$$
$$8x - k = y$$

Substitute y in to find x in terms of k

$8x - y = k$ (1)
$2x + 3y = 10k$ (2)
$8x - k = y$ (3)

Find x and y in terms of k

$2x + 3(8x - k) = 10k$
$2x + 24x - 3k = 10k$
$26x - 3k = 10k$
$26x = 13k$
$x = \frac{1}{2}k$

$8x - y = k$
$4k - y = k$
$4k = k + y$
$3k = y$