YEAR 11F AND H
KNOWLEDGE ORGANISERS

CRAWSHAW MATHEMATICS

BLOCK: EXPRESSIONS

A14 - MANIPULATING EXPRESSIONS

YEAR 11F — EXPRESSIONS…

A14 - MANIPULATING EXPRESSIONS

Sparx Maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify algebraic expressions which includes collecting like terms, expanding brackets and factoring
- Use identities
- Form and solve equations and inequalities
- Represent numbers algebraically

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: On equation where both sides have variables that cause the same answer includes ≡
Linear: an equation or function that is the equation of a straight line
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Like and unlike terms

Like terms are those whose variables are he same

Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.
It is used to identify equivalent expressions

Collecting like terms
Only **like terms** can be combined

$4x + 5b - 2x + 10b$

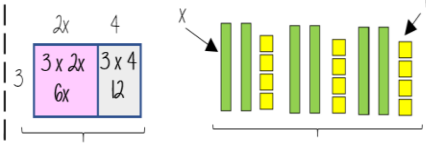
$2x + 15b$

Common misconceptions

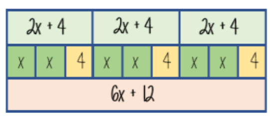
$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable x2 and x terms are un-like terms so can not be collected

Multiply single brackets $3(2x+4)$

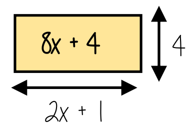


$6x + 12$ $6x + 12$



Different representations of $3(2x+4) = 6x + 12$

Factorise into a single bracket $8x + 4$



Try and make this the **highest** common factor

The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Algebraic numbers

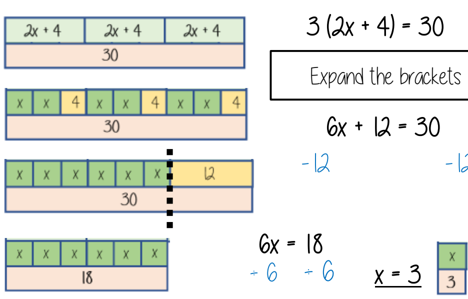
k is an odd number.
State whether each expression will be odd, even or could be either.

$k-1$	$2k$	$2k+1$	$3k$
Even	Even	Odd	Odd

Prove that the sum of two consecutive integers is odd

Let n be an integer.
$n + 1$ is 1 greater than n
$n + n + 1 \equiv 2n + 1$
Even

Solve equations with brackets



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$
$-12 \quad -12$

$6x = 18$
$\div 6 \quad \div 6$

$\underline{x = 3}$

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow -3 \leftarrow -2 \leftarrow 11$

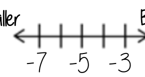
$x > 3$

Inequalities with negatives

Method 1: Make x positive first

$2 - 3x > 17$
$+3x \quad +3x$
$2 > 17 + 3x$
$-17 \quad -17$
$-15 > 3x$
$\div 3 \quad \div 3$
$\underline{-5 > x}$

x is true for any value smaller than -5



CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Method 2: Keep the negative x

$2 - 3x > 17$
$-2 \quad -2$
$-3x > 15$
$\div -3 \quad \div -3$
$x > -5$

x is true for any value bigger than -5

This cannot be true…

$\underline{x < -5}$

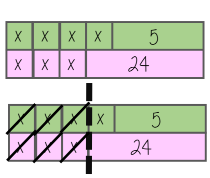
When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

$4x + 5 = 3x + 24$
$-3x \quad -3x$

$x + 5 = 24$
$-5 \quad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$
$5x + 20 < 3x + 6$
$2x + 20 < 6$
$2x < -14$
$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$
$5(-4) < 3(-6)$
$-20 < -18$
-20 IS smaller than -18

Formulae and Equations

Substitute in values

Formulae – all expressed in symbols → **Equations** – include numbers and can be solved

YEAR 11H – EXPRESSIONS…

A14 - MANIPULATING EXPRESSIONS

Sparx Maths

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Simplify algebraic expressions which includes collecting like terms, expanding brackets and factoring
- Use identities
- Form and solve equations and inequalities
- Represent numbers algebraically
- Work with Algebraic fractions

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
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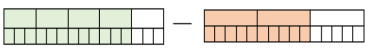
Identity: On equation where both sides have variables that cause the same answer includes $\equiv$
Linear: an equation or function that is the equation of a straight line
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Add and subtract algebraic fractions

H

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

Use equivalent fractions to find a common multiple for both denominators

$$\frac{2}{a} + \frac{1}{b}$$

$\times b$ $\quad$ $\times a$

$$\frac{2b}{ab} + \frac{1a}{ab}$$

This cant simplify any further… we could combine into one fraction though to get: $\frac{2b + 1a}{ab}$

Algebraic fractions use the same rules as basic fractions, We can only add or subtract things that are the same size (in the same **denomination**).

$$\frac{3x}{4} + \frac{2x}{3}$$

$\times 3$ $\quad$ $\times 4$

$$\frac{9x}{12} + \frac{8x}{12}$$

$$= \frac{9x + 8x}{12}$$

$$= \frac{17x}{12}$$

$$\frac{3}{x+1} + \frac{1}{2}$$

$\times (2)$ $\quad$ $\times (x+1)$

$$\frac{(3)(2)}{(x+1)(2)} + \frac{(1)(x+1)}{(2)(x+1)}$$

$$\frac{6}{2x+2} + \frac{x+1}{2x+2}$$

$$= \frac{6 + (x+1)}{2x+2} = \frac{x+7}{2x+2}$$

You might get questions which are a bit more complicated, it is still the same process though, All you are trying to do is to get the denominators the same…. think cross multiply if you're struggling

$$\frac{x+1}{x+2} + \frac{x+3}{x+4}$$

$$\frac{(x+1)(x+4)}{(x+2)(x+4)} + \frac{(x+3)(x+2)}{(x+4)(x+2)}$$

$$\frac{x^2+5x+4}{x^2+6x+8} + \frac{x^2+5x+6}{x^2+6x+8}$$

$$\frac{2x^2+10x+10}{x^2+6x+8}$$

Dividing any fractions

Remember to use reciprocals

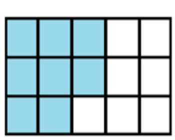
R

$$\frac{2}{5} \div \frac{3}{4}$$

$$\frac{2}{5} \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

Multiplying any fractions

R

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$

Multiply numerators together
and
Multiply denominators together

Multiply and divide algebraic fractions

To help, we can **factorise & cancel** *before* multiplying

$$\frac{7a-21}{a} \times \frac{3}{4a-12} = \frac{7 \times (a-3) \times 3}{a \times 4 \times (a-3)}$$

$$= \frac{7 \times 3}{a \times 4} = \frac{21}{4a}$$

Simplify:

$$\frac{5x+10}{3} \times \frac{x}{x+2}$$

$$= \frac{(5x+10) \times (x)}{(3) \times (x+2)}$$

$$= \frac{5x^2+10x}{3x+6}$$

$$= \frac{(5x)(x+2)}{(3)(x+2)}$$

common factor: $x+2$

$$= \frac{5x}{3}$$

Simplify:

$$\frac{4}{3x+9} \div \frac{3}{2x+6}$$

$$\frac{4}{3x+9} \times \frac{2x+6}{3}$$

$$= \frac{(4) \times (2x+6)}{(3x+9) \times (3)}$$

$$= \frac{8x+24}{9x+27}$$

$$= \frac{(8)(x+3)}{(9)(x+3)}$$

common factor: $x+3$

$$= \frac{8}{9}$$

YEAR 11F AND H
KNOWLEDGE ORGANISERS

BLOCK: GRAPHS

A15 - GRADIENTS AND LINES

A16 - NON-LINEAR GRAPHS

R7 - USING GRAPHS

YEAR 11F — GRAPHS...

A15 - GRADIENTS AND LINES

Sparx Maths

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Plot straight line graphs and interpret in the form y=mx+c
- Find the equation of straight lines from graphs and from coordinates.
- Work with lines Parallel to an axis
- Solve linear simultaneous equations graphically

Keywords

Coordinate: a set of values that show an exact position.
Horizontal: a straight line from left to right (parallel to the x axis)
Vertical: a straight line from top to bottom (parallel to the y axis)
Origin: (0,0) on a graph. The point the two axes cross
Parallel: Lines that never meet
Gradient: The steepness of a line
Intercept: Where lines cross

CRAWSHAW MATHEMATICS

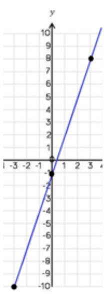
Plotting straight line graphs R

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

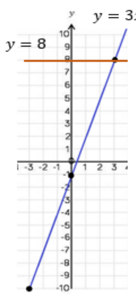
Remember to join the points to make a line

Find solutions graphically

For linear equations there is only one point the graph meets the x value

$x = 2$
$y = 4$

These two lines will cross at (2,4) because they are just x= and y= they are parallel to axes and meet in one place



$3x - 1 = 8$

Remember equation of a line format is y = mx + c

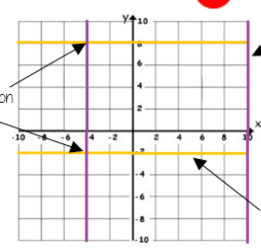
The solution is the point the two lines meet (3,8)

Lines parallel to the axes R

All the points on this line have a x coordinate of 10

'a' can be ONY positive or negative value including 0

Intersection points



Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

y = mx + c R

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$$y = mx + c$$

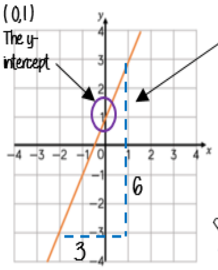
The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

The equation of a line can be rearranged: Eg:
y = c + mx
c = y — mx
Identify which coefficient you are identifying or comparing

Find the equation from a graph R

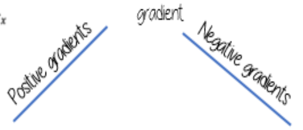
(0,1) The y-intercept



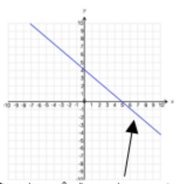
The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient



Lines with negative gradients

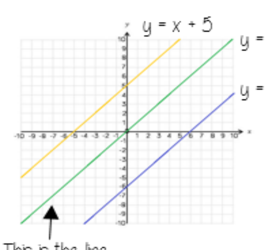


Any straight-line graph with a negative x value has a negative gradient

Eg y = -2x
y = -x   y + x = 12

Direction of all negative gradients

Lines in the form y = mx + c R



All the lines are **parallel** because the gradients are the same

This is the line y=x when the y and x coordinate are the same

y = mx + c

m represents the gradient

This shows the translation of that line. **eg** y = x + 5 Is the line y=x moved 5 places up the graph

5 has been added to each of the x coordinates

It's y axis intercept!

Equation from two points

Work out the equation of the line that passes through the points $(3, 5)$ and $(6, 14)$

$x_1\ y_1$ $\quad x_2\ y_2$

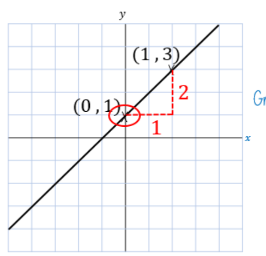
Gradient $= \frac{\text{Change in } y}{\text{Change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{14-5}{6-3} = \frac{9}{3}$$

$m = 3$

Equation of a straight line

Work out the equation of the straight line.



$c = 1$

Gradient $= \frac{\text{Change in } y}{\text{Change in } x} = \frac{2}{1}$

$m = 2$

$y = 2x + 1$

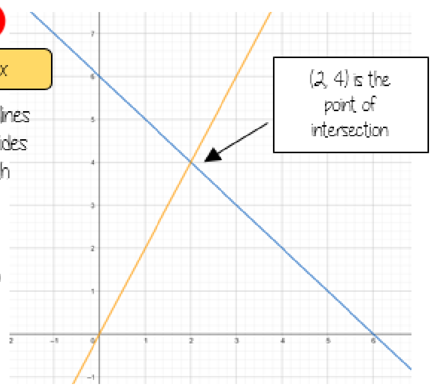
Solve graphically R

x + y = 6   y = 2x

Linear equations are straight lines. The point of intersection provides the x and y solution for both equations

The solution that satisfies both equations is

$x = 2$ and $y = 4$



(2, 4) is the point of intersection

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

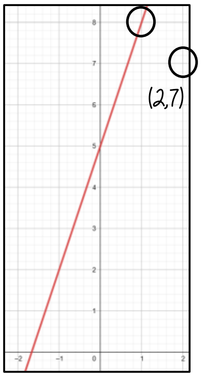
$$y = 3x + 5$$
$$8 = 3(1) + 5$$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5

Is (2,7) on the same line?

$$7 \neq 3(2) + 5$$

No 7 does NOT equal 6+5



YEAR 11H — GRAPHS...

A15 - GRADIENTS AND LINES

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Plot straight line graphs and interpret in the form y=mx+c
- Find the equation of straight lines from graphs and from coordinates.
- Work with lines Parallel and perpendicular to an axis
- Solve linear simultaneous equations graphically

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Gradient: The steepness of a line
Intercept: Where lines cross

Reciprocals R

The reciprocal of a number is the number you would have to multiply it by to get the answer 1.

4 The reciprocal is $\frac{1}{4}$

$\frac{2}{3}$ The reciprocal is $\frac{3}{2}$

0.25 $\frac{1}{4}$ The reciprocal is 4

Write the decimal as a fraction first!

The product of the gradients of a pair of perpendicular lines will always be -1 therefore you need to find the negative reciprocal

-3 The negative reciprocal is $\frac{1}{3}$

5 The negative reciprocal is $-\frac{1}{5}$

y = mx + c R

The **coefficient** of x (the number in front of x) tells us the gradient of the line

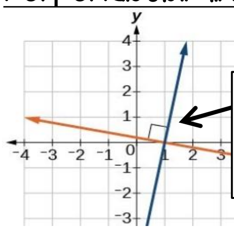
$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**

Perpendicular lines H

Unlike parallel lines, **perpendicular lines** do intersect. Their intersection forms a right or 90-degree angle.



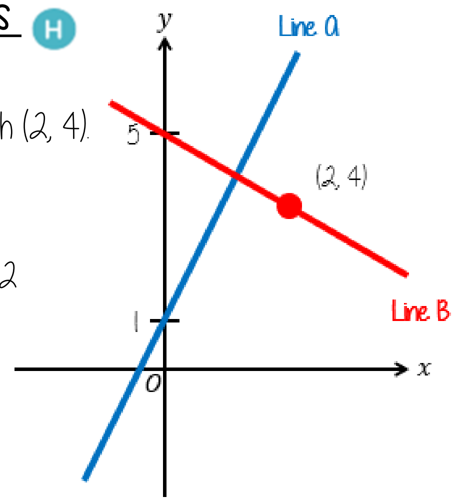
The two lines are perpendicular.

The slope of one line is the negative reciprocal of the slope of the other line. The product of a number and its reciprocal is 1.

Equations for Perpendicular lines H

Example 1 Line a $y = 2x + 1$

Line B is perpendicular and passes through (2, 4). Find the equation of Line B.



$$y = mx + c$$

gradient (m) = negative reciprocal of 2

$$y = -\frac{1}{2}x + c$$

Substitute (2,4) to find c

$$4 = -(0.5 \times 2) + c$$
$$4 = -1 + c$$
$$5 = c$$

$\boxed{y = -\frac{1}{2}x + 5}$ Or $\boxed{y = 5 - \frac{1}{2}x}$

Example 2

Line L_2 is perpendicular to $y = 1 - \frac{1}{3}x$ and passes through (5, 7).

Find the equation of Line L_2.

Gradient is the negative reciprocal of this line

$y = mx + c \longrightarrow y = 3x + c$

substitute x & y which are (5, 7) to find c

$$7 = (3 \times 5) + c$$
$$7 = 15 + c$$
$$-8 = c$$

Line L_2: $\boxed{y = 3x - 8}$

YEAR 11F – GRAPHS...

A16 - NON-LINEAR GRAPHS

Sparx Maths

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Plot and read quadratic graphs
- Plot and read cubic and reciprocal graphs
- Recognise graph shapes
- Identify and interpret roots and intercept of quadratics

Keywords

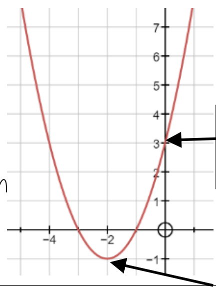
Quadratic: a curved graph with the highest power being 2. Square power.
Reciprocal: a reciprocal is 1 divided by the number
Cubic: a curved graph with the highest power being 3. Cubic power.
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

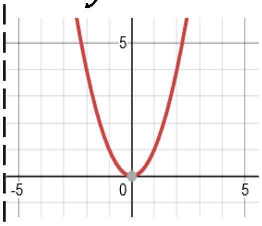
Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

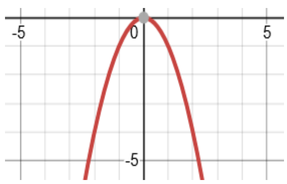
When an quadratic inequality is solved it provides the **range of values** that are possible.
When an equation has **solutions greater than 0** then the solutions are taken from **above the x axis**.
When an equation has solutions **less than 0** then the solutions are taken from **below the x axis**.

A quadratic graph will always be in the shape of a parabola.

$$y = x^2$$

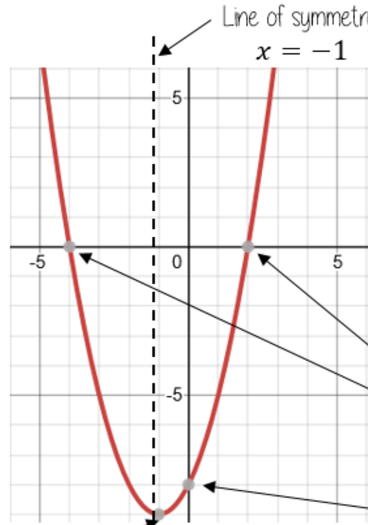


$$y = -x^2$$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Line of symmetry $x = -1$



Turning point $(-1, -9)$

Examples

$$y = x^2 + 2x - 8$$

A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation.
This is dependant on how many times the graph crosses the x axis.

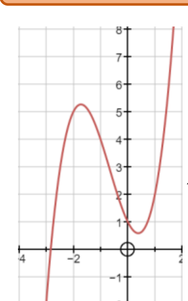
Roots $x = -4$
$x = 2$

y intercept = -8

Interpret other graphs

Cubic Graphs

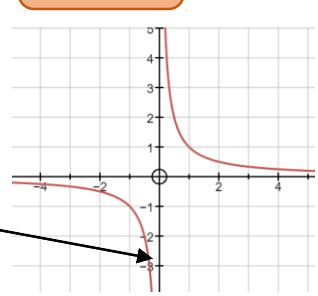
$y = x^3 + 2x^2 - 2x + 1$



If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$$y = \frac{1}{x}$$

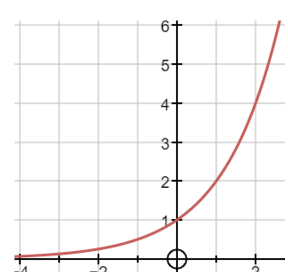


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x



YEAR 11H – GRAPHS...

A16 - NON-LINEAR GRAPHS

Sparx Maths

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Plot and read quadratic graphs
- Plot and read cubic and reciprocal graphs
- Recognise graph shapes
- Identify and interpret roots and intercept of quadratics
- Find the equation of a circle
- Find the equation of the tangent to any curve

Keywords

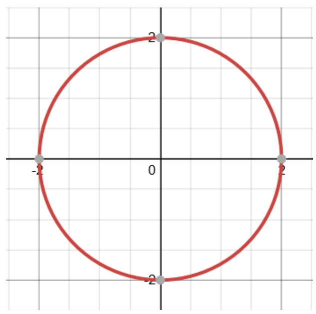
Quadratic: a curved graph with the highest power being 2. Square power.
Reciprocal: a reciprocal is 1 divided by the number
Cubic: a curved graph with the highest power being 3. Cubic power.
Origin: the coordinate (0, 0)
Parabola: a 'U' shaped curve that has mirror symmetry

Equations of a Circle (H)

The **equation of a circle** will be in the format:

$$\boxed{x^2 + y^2 = radius^2}$$

The **centre** of each circle will be at the coordinate **(0,0)**.



$$x^2 + y^2 = 4$$
$$Radius = \sqrt{4}$$
$$= \pm 2$$

Therefore we can plot the following coordinates to support us sketching our graph: (0,2), (0,-2), (2,0), (-2,0)

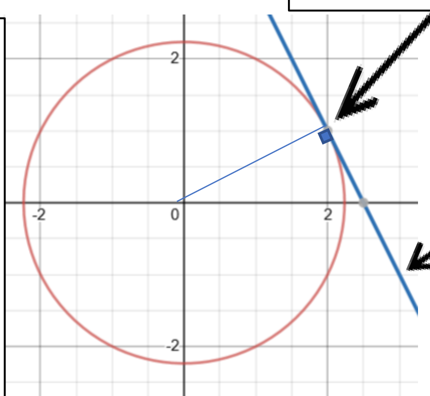
Tangent to a Circle (H)

A **tangent** touches a circle at **one point**.

Find the equation of the tangent to the circle with equation:

$$x^2 + y^2 = 5$$

which passes through the point (2,1)



A **tangent** line is **perpendicular** to the **radius** of the circle.

The gradient of the tangent is the **negative reciprocal** of the gradient of the equation of the line of the radius.

Step 1: Find the equation of the line which is the radius of the circle.

$$gradient = \frac{1}{2}$$

$$therefore\ y = \frac{1}{2}x$$

Step 2: The tangent is perpendicular to the radius.

$$gradient\ of\ tangent = negative\ reciprocal\ of\ \frac{1}{2}\ \boxed{= -2}$$

$$y = -2x + c$$

When x = 2 and y = 1 from the coordinate (2,1)

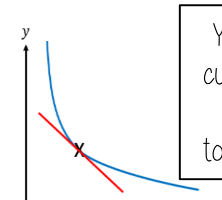
Step 3: Substitute in the given coordinate (2,1) in to $y = -2x + c$ to find c

$$1 = (-2 \times 2) + c$$
$$1 + 4 = c$$
$$5 = c$$

Therefore the Equation of the tangent is :

$$\boxed{y = -2x + 5}$$

Tangent to a Curves (H)

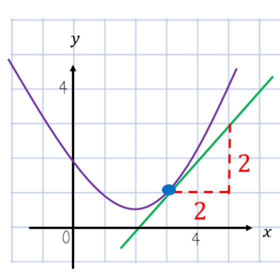


You can also get tangents on curves. Be careful though, make sure your tangent is only touching the curve at one point

The tangent to the curve at $(3, 1)$ has been drawn.
Now find the equation of the tangent. $y = mx + c$

$m = 1$ $\quad x = 3, y = 1$
$1 = 1 \times 3 + c$
$1 = 3 + c$
$c = 1 - 3$
$c = -2$

Equation of the tangent is
$y = x - 2$



YEAR 11F — GRAPHS...

R7- USING GRAPHS

Sparx Maths — **Extension work** – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Reflect shapes in given lines
- Construct and interpret conversation and real life graphs
- Construct and interpret speed/distance and time graphs
- Recognise and interprets direct and inverse proportion

Keywords

Convert: change

Gradient: The steepness of a line

Intercept: Where lines cross

Origin: the coordinate (0, 0)

Substitute: putting numbers where letters are – replacing numbers into a formula

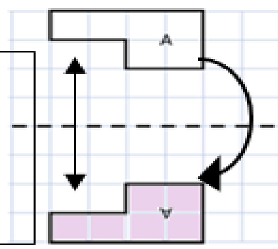
Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

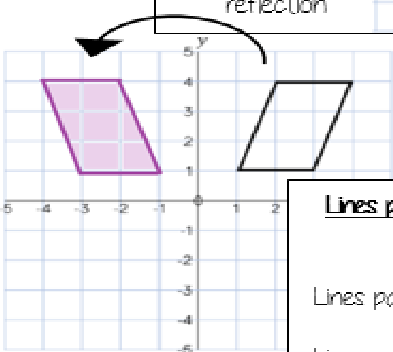
Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Reflect horizontally/ vertically

R

All points need to be the same distance away from the line of reflection





Reflection in the line y axis – this is also a reflection in the line x=0

Lines parallel to the x and y axis

REMEMBER

Lines parallel to the x-axis are y = ____

Lines parallel to the y-axis are x = ____

Speed, Distance, Time

R

minutes $\xrightarrow{\div 60}$ hours

hours $\xrightarrow{\times 60}$ minutes

Before calculations – make sure you are working in the same units as the speed

Learn or learn how to rearrange the formular for speed, distance and time

Substitute in the variables given

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{distance} = \text{speed} \times \text{time}$$

Distance – Time graphs

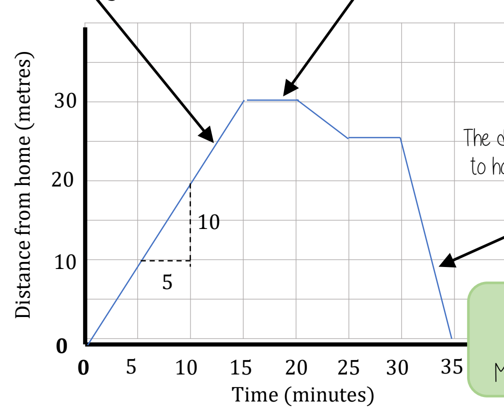
R

$$\text{Gradient} = \text{speed}$$

The steeper a gradient the faster the speed

$\frac{10}{5} = 2$ metres per min

Horizontal lines represent staying still

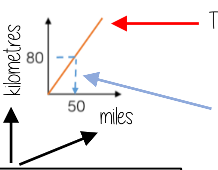


The distance coming closer to home shows the return journey

Units are important. Meters per minute

Conversion Graphs

R



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.

Using a ruler helps for accuracy

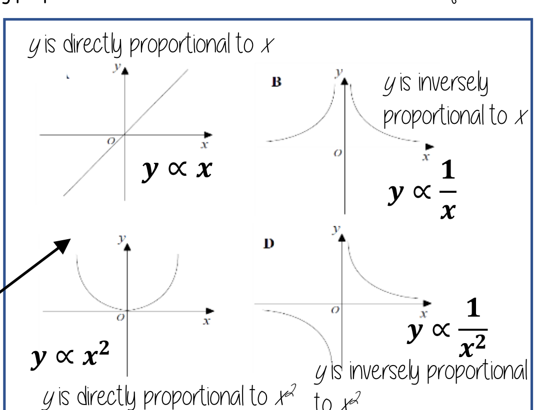
Showing your conversion lines help as a "check" for solutions

Direct and inverse proportion

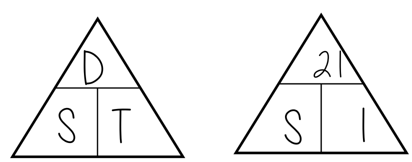
Variables are **directly proportional** when the **ratio is constant** between the quantities.

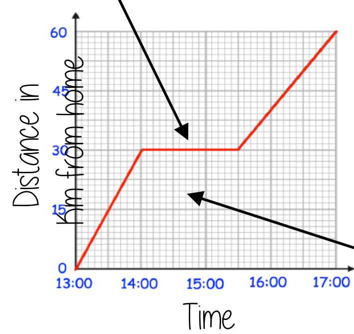
Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing**

Direct and inverse proportion can also be represented on graphs



Horizontal sections are where the object is stationary





$$Speed = \frac{distance}{time}$$

$$Speed = \frac{21}{1}$$

$$Speed = 21km/h$$

Diagonal lines show the object moving away from home or moving closer to home

YEAR 11H – GRAPHS…

R7- USING GRAPHS

Extension work – Codes for related Independent Learning tasks on SPARX maths:

Sparx Maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Reflect shapes in given lines
- Construct and interpret conversation and real life graphs
- Construct and interpret speed/distance and time graphs
- Recognise and interprets direct and inverse proportion
- Estimate the area under a curve

Keywords

Convert: change

Gradient: The steepness of a line

Intercept: Where lines cross

Origin: the coordinate (0, 0)

Substitute: putting numbers where letters are – replacing numbers into a formula

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

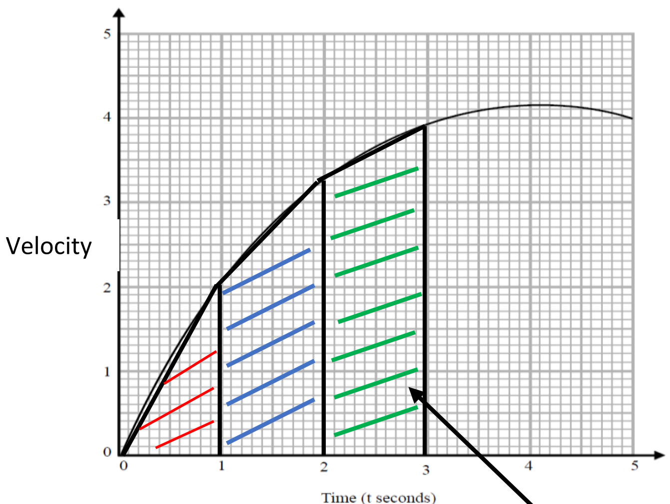
Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

area under a curve

H

A **velocity-time** graph (or speed-time graph) is a way of visually expressing a journey. With speed or velocity on the y-axis and time on the x-axis. It tells us **how someone's speed has changed over a period of time**.

The **distance** completed in the journey can be calculated from the **area underneath the curve**.



$$\left(\frac{1\times 2}{2}\right)+\left(\frac{(2+3.2)\times 1}{2}\right)+\left(\frac{(3.2+3.9)\times 1}{2}\right)$$

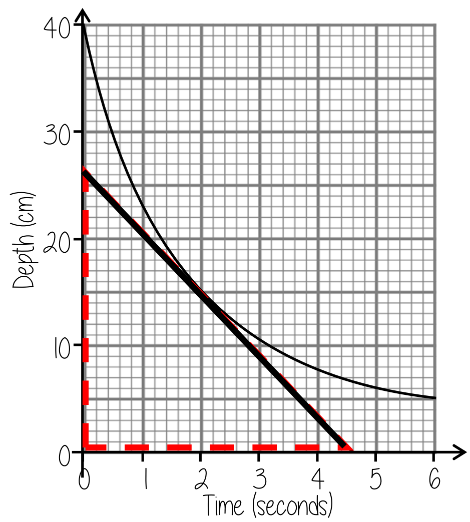
$$= 7.15m$$

Use 3 strips of equal width to find an estimate of the distance travelled in the first 3 seconds.

The strips will either be triangles or trapeziums. You will calculate the area of each section separately and combine the answers for the complete distance.

Rate of change using tangents

Estimating the Rate of Change Using the Gradient of a Tangent



Calculate the gradient of the tangent, remember this is: Gradient = $\frac{\text{Change in } y}{\text{Change in } x}$

Change in y is 26 and the Change in x is 4.6 so

At 2 seconds the rate of depth change = $\frac{26}{4.6}$

H ≈ 5.7 cm per second

Here is the graph $y = -x^3 + 3x + 4$ By drawing suitable trapezia, estimate the area under the curve between $x = -2$ and $x = 1$

H

Area $= \frac{1}{2} \times (6 + 2) \times 1$

$= 4$ units^2

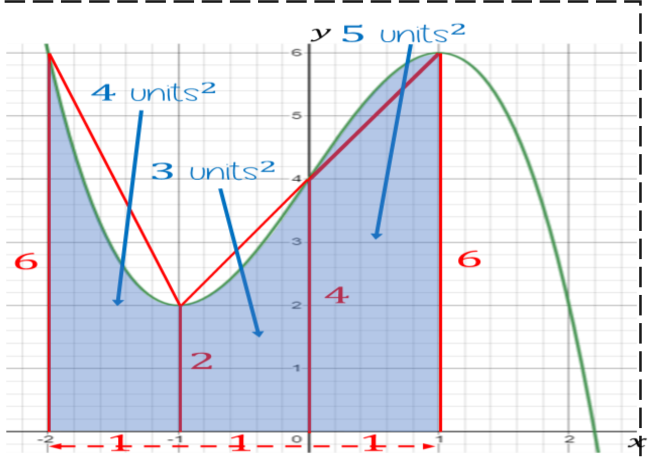
Area $= \frac{1}{2} \times (2 + 4) \times 1$

$= 3$ units^2

Area $= \frac{1}{2} \times (4 + 6) \times 1$

$= 5$ units^2

Total area under curve ≈ 12 units^2



YEAR 11F AND H
KNOWLEDGE ORGANISERS

BLOCK: ALGEBRA

A17 - EXPANDING AND FACTORISING

A18 - CHANGE THE SUBJECT

A19 - FUNCTIONS

YEAR 11F – ALGEBRA...

A17- EXPANDING AND FACTORISING

Sparx Maths

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Expand and Factorise
- Expand and factorise quadratics
- Solve quadratics equal to 0

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Linear: an equation or function that is the equation of a straight line
Quadratic: a curved graph with the highest power being 2 Square power

CRAWSHAW MATHEMATICS

Multiply single brackets

	2x	4
3	3 x 2x 6x	3 x 4 12

6x + 12

Single: Where each term inside the bracket is multiplied by the term on the outside of the bracket.

3 (2x + 4) = 6x + 12

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

x	x	x	x	x	x

$6x \times 1$ OR $6 \times x$

Factors of $6x$
$6, x, 1, 6x, 2x, 3, 3x, 2$

$2x \times 3$

$3x \times 2$

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

$3x, 6x, 9x$ …

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

This list continues and doesn't end

Non example of a multiple

4 . 5 is not a multiple of 3 because it is 3 x 1 . 5

Not an integer

Expanding double brackets

Double: Where each term in the first bracket is multiplied by all terms in the second bracket. A double bracket will be a quadratic equation.

$(p+2)(2p-1)$ $= 2p^2 + 4p - p - 2$
$= 2p^2 + 3p - 2$

$(p+2)^2 = (p+2)(p+2)$ $= p^2 + 2p + 2p + 4$
$= p^2 + 4p + 4$

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18	1, 2, 3, 6, 9, 18
30	1, 2, 3, 5, 6, 10, 15, 30

Common factors (factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Solve when = 0

solve $3x + 4 = 0$
$\quad -4 \quad -4$

$\boxed{-4 + 4 = 0}$

So $3x = -4$
$\div 3 \quad \div 3$
$x = \frac{-4}{3}$

Solve the equation $(2x+1)(1-x) = 0$

$\boxed{(2x+1)}\boxed{(1-x)} = 0$

Work with both solution separately

$2x + 1 = 0$
$-1 \quad -1$
$2x = -1$
$\div 2 \quad \div 2$
$x = \frac{-1}{2}$

$1 - x = 0$
$+x \quad +x$
$x = 1$

Factorise and solve:

$x^2 + 4x - 5 = 0$
$(x-1)(x+5) = 0$

Therefore the solutions are:

Either $x - 1 = 0$
$x = 1$

Or $x + 5 = 0$
$x = -5$

Factorise into a single bracket $8x + 4$

	8x + 4
4	2x + 1

Try and make this the **highest** common factor

The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Factorising Quadratics

Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Factorise:

Add to find the middle term 2+4

$x^2 + 6x + 8$

Multiply to find the end term ~~1 8~~ 2 4

$= (x+2)(x+4)$

Add to find the middle term -3 + 1

$x^2 - 2x - 3$

Multiply to find the end term 1 3

$= (x-3)(x+1)$

Algebraic constructs

Expression
A sentence with a minimum of two numbers and one maths operation

Equation
A statement that two things are equal

Term
A single number or variable

Identity
An equation where both sides have variables that cause the same answer includes ≡

Formula
A rule written with all mathematical symbols
e.g. area of a rectangle
A = b x h

YEAR 11H – ALGEBRA…

A17- EXPANDING AND FACTORISING

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Expand and Factorise
- Expand and factorise quadratics
- Solve quadratics equal to 0
- Factorise complex expressions
- Solve quadratics by completing the square
- Solve quadratics by using the formula

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Linear: an equation or function that is the equation of a straight line
Quadratic: a curved graph with the highest power being 2 Square power

Solving Quadratics

Quadratics are always in the form:

$$ax^2 + bx + c = 0$$

We can solve quadratic equations in 4 different ways

1. Factorising – put into brackets first

2. Completing the square

$$\boxed{\left(x+\frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2 = 0}$$

3. Quadratic formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

4. Graphically (see Simultaneous Equation (Y10) knowledge organisers)

Factorising Quadratics to solve

Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Factorise:

Add to find the middle term 2+4

$$x^2 + 6x + 8$$

Multiply to find the end term ~~1 8~~ 2 4

$$= (x+2)(x+4)$$

Add to find the middle term -3 + 1

$$x^2 - 2x - 3$$

Multiply to find the end term 1 3

$$= (x-3)(x+1)$$

Factorise and solve:

$$x^2 + 4x - 5 = 0 \qquad (x-1)(x+5) = 0$$

Therefore the solutions are:

Either $x - 1 = 0$ $\quad x + 5 = 0$

$x = 1$ Or $x = -5$

Factorising and solving with Coefficients

H

Solve: $\underset{a}{6x^2} + \underset{b}{7x} - \underset{c}{3} = 0$

Find two numbers…

Product = ac = –18 $\quad -2 \;\&\; 9$

Sum = b = 7

	$3x$	-1
$2x$	$6x^2$	$-2x$
$+3$	$+9x$	-3

$$(2x+3)(3x-1) = 0$$

$(2x+3) = 0 \qquad (3x-1) = 0$

$2x = -3 \qquad 3x = 1$

$x = -\frac{3}{2}$ And $x = \frac{1}{3}$

Completing the square

Completing the square is a method used to solve quadratic equations that will not factorise.

$$\underbrace{x^2 + 4x}\underbrace{- 15} = 0$$

$$(x+2)^2 \qquad -15 = 0$$

$$= (x+2)(x+2)$$

$$= x^2 + 4x + 4$$

We don't want this extra

$$(x+2)^2 - 4$$

$$\underbrace{(x+2)^2 - 4}\underbrace{- 15} = 0$$

$$(x+2)^2 - 19 = 0$$

H We can solve quadratic using Completing the Square:

$$x^2 - 4x + 1 = 0$$

$$(x-2)^2 + 1 = 0$$

$$(x-2)^2 - 4 + 1 = 0$$

$$(x-2)^2 - 3 = 0$$

$$(x-2)^2 = 3$$

Rearrange to get x on its own.

$$x - 2 = \pm\sqrt{3}$$

$$x = \pm\sqrt{3} + 2$$

$$x = \pm 1.7\ldots + 2$$

Quadratic formula

When a quadratic doesn't factorise or is difficult to use completing the square method due to the value of its coefficients we use the quadratic formula:

H

$$\boxed{ax^2 + bx + c = 0} \qquad \boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

Solve:

$$x^2 - 5x + 2 = 0$$

$$a = +1 \qquad b = -5 \qquad c = +2$$

Watch out for double negatives with your 'b' value! If you are using a calculator remember to add brackets!

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - (4 \times (+1) \times (+2))}}{2 \times (1)}$$

$$x = \frac{5 \pm \sqrt{25-8}}{2}$$

$$x = \frac{5+\sqrt{17}}{2} \qquad x = \frac{5-\sqrt{17}}{2}$$

$$x = 4.6 \qquad x = 0.4$$

YEAR 11F – ALGEBRA

A18 - CHANGING THE SUBJECT

Sparx Maths

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve linear equations
- Solve inequalities
- Form and solve equations and inequities
- Change the subject

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

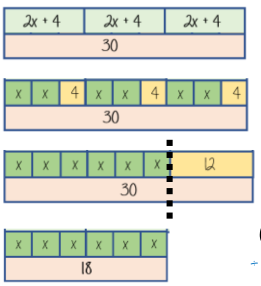
Equation: an equation says that two things are equal – it will have an equals sign =

Linear: an equation or function that is the equation of a straight line

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations with brackets

R



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$\div 6 \quad \div 6$

$\underline{x = 3}$

Form and solve inequalities

R

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow \div 3 \leftarrow -2 \leftarrow 11$

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \quad +3x$

$2 > 17 + 3x$

$-17 \quad -17$

$-15 > 3x$

$\div 3 \quad \div 3$

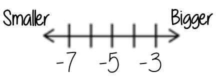
$\underline{-5 > x}$

x is true for any value smaller than -5

✓ CHECK IT!

$2 - 3(-6) = 20$

TRUE/ CORRECT



Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \quad -2$

$-3x > 15$

$\div -3 \quad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

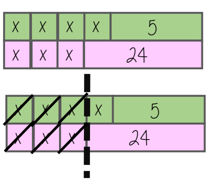
$4x + 5 = 3x + 24$

$-3x \quad -3x$

$x + 5 = 24$

$-5 \quad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

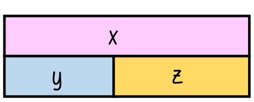
✓ -20 IS smaller than -18

Formulae and Equations

Substitute in values

Formulae – all expressed in symbols → **Equations** – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

$\longrightarrow +z \longrightarrow x$

$\longleftarrow -z \longleftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \quad +3$

$4x = 12$

$\div 4 \quad \div 4$

$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$

$+s \quad +s$

$xy = a + s$

$\div y \quad \div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

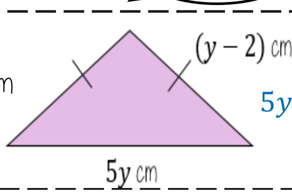
e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \dfrac{4x + 9}{2}$

Form and Solve equations

The perimeter of this triangle is 45 cm

Write an equation to represent this information and Solve.



Perimeter

$5y + y - 2 + y - 2 \equiv 7y - 4$

$7y - 4 = 45$

Solve the equation to find the value of y.

$7y - 4 = 45$

$7y = 49$

$y = 7$

YEAR 11H — ALGEBRA

A18 - CHANGING THE SUBJECT

Extension work – Codes for related Independent Learning tasks on SPARX maths:

Sparx Maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve linear equations
- Solve inequalities
- Form and solve equations and inequities
- Change the subject where the subject appears more than once
- Solve equations by iteration

Keywords

Solution: a value we can put in place of a variable that makes the equation true

Variable: a symbol for a number we don't know yet.

Equation: an equation says that two things are equal – it will have an equals sign =

Linear: an equation or function that is the equation of a straight line

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.

Solve equations by iteration

H

Calculate the values of x_1, x_2, x_3 to find an estimate for the solution to $x^3 + 3x = 2$

Core knowledge

Iteration is the **repetition** of a mathematical procedure applied to the result of a previous application, typically as a means of **obtaining successively closer approximations** to the solution of a problem.

$$x_{n+1} = \frac{2}{x_n^2 + 3}$$ When $x_0 = 0$

$$x_{0+1} = \frac{2}{0^2 + 3} = 0.\dot{6}$$

We substitute this value into the next step.

$$x_{1+1} = \frac{2}{0.\dot{6}^2 + 3} = 0.5806451613$$

$$x_{2+1} = \frac{2}{(0.58\ldots)^2 + 3} = 0.5993140006$$

An estimate of the solution is 0.6 because all of the solutions round to 1dp.

NOTE: Make sure ALL working out is fully wrote on a GCSE paper

Change the subject

H

Make v the subject of the formula

$$g = \frac{13(d - 3v)}{v}$$

get rid of the fraction first

$$vg = 13(d - 3v)$$

$$vg = 13d - 39v$$

$$vg + 39v = 13d$$

Factorise at this point to get the coefficient on its own

$$v(g + 39) = 13d$$

Rearrange to get the coefficient on its own

$$v = \frac{13d}{(g + 39)}$$

$$v = \frac{13d}{(g + 39)}$$

Iteration in context

H

The number of people living in a town t years from now is P_t where

$$P_0 = 55000$$
$$P_{t+1} = 1.03(P_t - 800)$$

Work out the number of people in the town 3 years from now.

$$P_1 = 1.03\,(55000 - 800) = 55826$$

$$P_2 = 1.03\,(ANS - 800) = 56677$$

$$P_3 = 1.03\,(ANS - 800) = 57553$$

After 3 years there will be 57553 people living in the village

All rounded to the nearest integer

YEAR 11F – ALGEBRA…

A19- FUNCTIONS

Extension work – Codes for related Independent Learning tasks on SPARX maths:

Sparx Maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use function machines
- Substitute into expressions and formulae
- Graphs of quadratic functions
- Understand and use trigonometric functions

Keywords

Function: a relationship that instructs how to get from an input to an output.
Input: the number/ symbol put into a function.
Output: the number/ expression that comes out of a function.
Operation: a mathematical process
Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
Quadratic: a curved graph with the highest power being 2. Square power.
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

Single function machines (algebra)

INPUT → × 10 → OUTPUT

a → 10a

3c → 30c

÷ 10

To find the input from the output

Use the **INVERSE** operation

Two step function machines

INPUT → ☐ → ☐ → OUTPUT

Calculate the value at the end of each operation

For the input use the **INVERSE** operations

Substituting known variables

A line has the equation 3x + y = 14

Stephanie knows the point x = 4 lies on that line. Find the value for y.

Two different variables, two solutions

$x = 4$

$3x + y = 14$

x	x	x	y
14			

$3(4) + y = 14$

4	4	4	y
14			

$12 + y = 14$
$-12 \quad -12$
$y = 2$

y
2

Substituting in an expression

Substitute 2y in place of the x variable as they represent the same value

x = 2y

y	y
x	

x + y = 30

x	y
30	

$x = 2y$ $\quad x + y = 30$

y	y	y
30		

$3y = 30$

y
10

$3y = 30$
$\div 3 \quad \div 3$
$y = 10$

$x = 2y$

10	10
x	

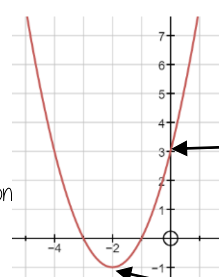
$x = 20$

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Intersection with the y axis

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

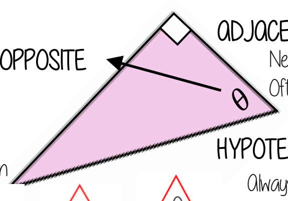
Coordinate pairs for plotting $(-3, 0)$

Plot all of the coordinate pairs and join the points with a curve (freehand)

Quadratic graphs are always symmetrical with the turning point in the middle

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



Always opposite an acute angle
Useful to label second
Position depend upon the angle in use for the question

OPPOSITE

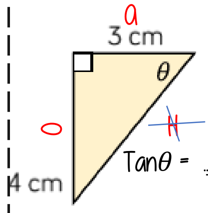
ADJACENT
Next to the angle in question
Often labelled last

HYPOTENUSE
Always the longest side…opposite the right angle
Useful to label this first

REMEMBER: O / S × H   A / C × H   O / T × A

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trig ratio
Substitute values into the ratio formula

θ = Sin^{-1} $\frac{\text{opposite}}{\text{hypotenuse}}$

θ = Cos^{-1} $\frac{\text{adjacent}}{\text{hypotenuse}}$

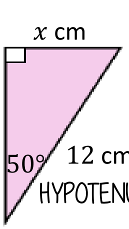
θ = Tan^{-1} $\frac{\text{opposite}}{\text{adjacent}}$

Tanθ = $\frac{3}{4}$

θ = Tan^{-1} $\frac{3}{4}$

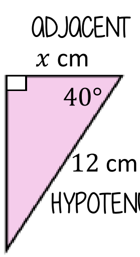
θ = 36.9°

Sin and Cos ratio: side lengths



Sinθ = $\frac{\text{opposite}}{\text{hypotenuse}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio



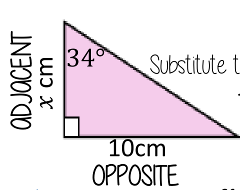
Cosθ = $\frac{\text{adjacent}}{\text{hypotenuse}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Tangent ratio: side lengths

Tanθ = $\frac{\text{opposite}}{\text{adjacent}}$



Substitute the values into the tangent formula

Tan34 = $\frac{10}{x}$

x × Tan34 = 10

x = $\frac{10}{\text{Tan34}}$ = 14.8cm

YEAR 11H – ALGEBRA…

A19- FUNCTIONS

Extension work – Codes for related Independent Learning tasks on SPARX maths:

Sparx Maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use function machines
- Substitute into expressions and formulae
- Graphs of quadratic functions
- Understand and use trigonometric functions

Keywords

Function: a relationship that instructs how to get from an input to an output.
Input: the number/ symbol put into a function.
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Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
Quadratic: a curved graph with the highest power being 2. Square power.
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

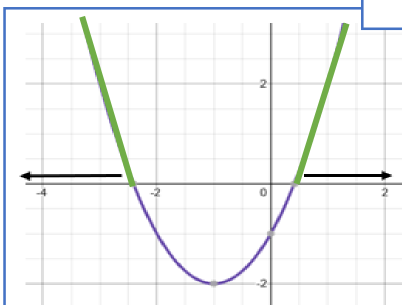
Quadratic inequalities H

When an quadratic inequality is solved it provides the **range of values** that are possible.

Remember: you have to sketch the quadratic first, so solve the quadratic by factorising (put into brackets first) or using the formula:

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$x^2 + 2x - 1 \geq 0$

The range of solutions are:

$x \geq 0.5 \text{ and } x \leq -2.5$

When an equation has **solutions greater than 0** then the solutions are taken from **above the x axis**.

$x^2 + 3x + 2 \leq 0$

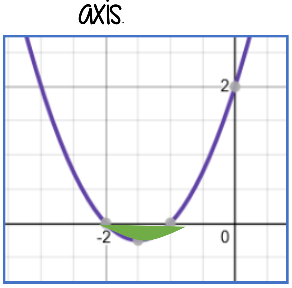
factorised is:

$(x+1)(x+2) \leq 0$

The range of solutions are

$-2 \leq x \leq -1$

When an equation has solutions **less than 0** then the solutions are taken from **below the x axis**.



Composite functions

We need to **substitute** one function into another and simplify.

$f(x) = 3x$

$v(x) = 2x + 5$

Substitute f into v

$vf(x) = ?$

$vf(x) = 2(3x) + 5$

Simplify the expression

$vf(x) = 6x + 5$

$q(x) = \dfrac{x+3}{2}$

$s(x) = 3x - 5$

$sq(x) = ?$

Substitute q into s

$sq(x) = 3\left(\dfrac{x+3}{2}\right) - 5$

$sq(x) = \dfrac{3x+9}{2} - 5$

H

Inverse functions H

$f(x) = \dfrac{x}{5} - 4$

We need to **reverse** the function machine and use the **inverse** operations.

$\div 5 \rightarrow \; - 4 \rightarrow$

$\leftarrow \times 5 \; \leftarrow + 4$

$f^{-1}(x) = \dfrac{X + 4}{5}$

Given $f(x) =$ find $f^{-1}(x)$

$f(x) = \dfrac{x-2}{5}$

$x \xrightarrow{-2} x - 2 \xrightarrow{\div 5} \dfrac{x-2}{5}$

$5x + 2 \xleftarrow{+2} 5x \xleftarrow{\times 5} x$

$f^{-1}(x) = 5x + 2$

Sometimes it might be a different letter e.g. g(X) it is still the same process

$g(x) = 3 - 2x$

Find an expression for $g^{-1}(x)$

$x \rightarrow \boxed{\times -2} \rightarrow \boxed{+3} \rightarrow f(x)$

$\dfrac{x-3}{-2} \xleftarrow{\div -2} x - 3 \xleftarrow{-3} x$

$g^{-1}(x) = \dfrac{3-x}{2}$

YEAR 11F AND H
KNOWLEDGE ORGANISERS

BLOCK: REASONING

N20 - MULTIPLICATIVE REASONING

G17 - GEOMETRIC REASONING

A20 - ALGEBRAIC REASONING

YEAR 11F – REASONING…

N20 - MULTIPLICATIVE REASONING

Extension work – Codes for related Independent Learning tasks on SPARX maths:

Sparx Maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use Scale factors
- Understand direct and inverse proportion
- Work with compound measures
- Work with Ratio problems

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Congruent: the same size and shape
Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios

CRAWSHAW MATHEMATICS

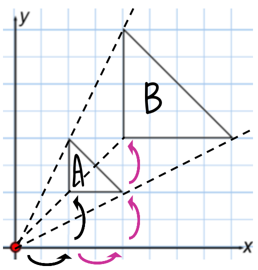
Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

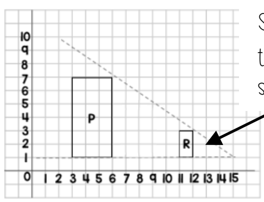
The distance from the point enlarges by 2



Fractional scale factors R

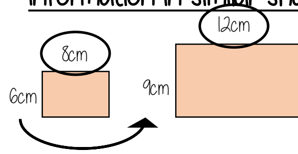
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15, 1)



SF: $\frac{1}{3}$ - R is three times smaller than P

Information in similar shapes



Compare the equivalent side on both shapes

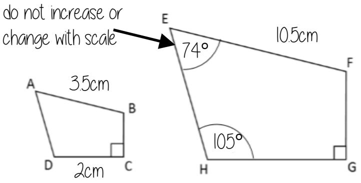
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

AB and EF are corresponding

Remember angles do not increase or change with scale



Direct Proportion R

As one variable changes the other changes at the same rate.

4 cans of pop = £2.40

4 cans of pop = £2.40
2 cans of pop = £1.20
(x 0.5)

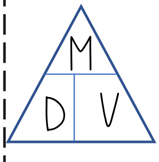
This multiplier is the same In the same way that this would be for ratio

This is a multiplicative change

4 cans of pop = £2.40
12 cans of pop = £7.20
(x 3)

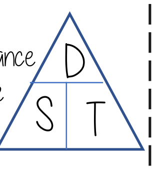
Sometimes this is easiest if you work out how much one unit is worth first
e.g. 1 can of pop = £0.60

Compound Measures



Density/Mass/Volume formula

Speed/Distance and Time formula



Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

	Shop A	Shop B
	4 cans for £1.20	3 cans for 93p
	£1.20 ÷ 4	£0.93 ÷ 3
Cost per item	1 can is £0.30 Or 30p	1 can is £0.31 Or 31p

Shop A is the best value as it is 1p cheaper per can of pop

	Shop A	Shop B
	4 cans for £1.20	3 cans for 93p
	4 ÷ £1.20	3 ÷ £0.93
Cost per pound	£1 buys 3.333 cans of pop	£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2   × 4

T	1	2	8
G	40	20	5

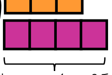
× 2   ÷ 4

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

James : Lucy
3 : 4
(x 50)
£150 : £200

James = 3 x £50 = £150



£350

Lucy = 4 x £50 = £200

Ratio and scale

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image	Real life
1cm	30cm
10cm	300cm

(x 10)

Ratios in 1:n and n:1

Show the ratio 4:20 in the ratio of 1:n

Cancel down until the part indicated represents 1

The question states that this part has to be 1 unit. Therefore Divide by 4

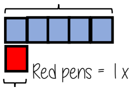
4 : 20
1 : 5

This side has to be divided by 4 too – to keep in proportion

Inside a box are blue and red pens in the ratio 5:1. If there are 10 red pens how many blue pens are there?

Blue : Red
5 : 1
(x 10)
50 : 10

Blue pens = 5 x 10 = 50 pens



Red pens = 1 x 10 = 10 pens

There are 50 Blue Pens

YEAR 11H – REASONING…

N20 - MULTIPLICATIVE REASONING

Sparx Maths

Extension work – Codes for related Independent Learning tasks on SPARX maths:

What do I need to be able to do?

By the end of this unit you should be able to:

- Use Scale factors
- Understand direct and inverse proportion
- Work with compound measures
- Work with Ratio problems

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
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Equivalent: of equal value
Proportion: a statement that links two ratios

Proportion – Key concept (H)

Variables are **directly proportional** when the **ratio is constant** between the quantities.

Variables are **inversely proportional** when **one quantity increases in proportion to the other decreasing.**

α is the symbol we use to show that one variable is in proportion to another.

Direct proportion: $\boxed{y \propto x}$

Inverse proportion: $\boxed{y \propto \frac{1}{x}}$

Direct proportion: (H)

g is directly proportional to the square root of h

When $g = 18, \quad h = 16$

Find the possible values of h when $g = 2$

Form an equation using **α** and **k** to show that one variable is in proportion to the other. Substitute the values and rearrange and solve to find the constant k.

$$g \propto \sqrt{h}$$
$$g = k\sqrt{h}$$
$$18 = k\sqrt{16}$$
$$18 = 4k$$
$$4.5 = k$$
$$g = 4.5\sqrt{h}$$

Substitute **k** back into the equation. Use the equation to answer the next part of the question

$$\boxed{g = 4.5\sqrt{h}}$$

Find h When $g = 2$

$$2 = 4.5\sqrt{h}$$
$$\frac{2}{4.5} = \sqrt{h}$$
$$\left(\frac{4}{9}\right)^2 = h$$
$$\frac{16}{81} = h$$

Inverse proportion: (H)

The time taken, t, for passengers to be checked-in is inversely proportional to the square of the number of staff, s, working.

Watch out for questions that are real life!!

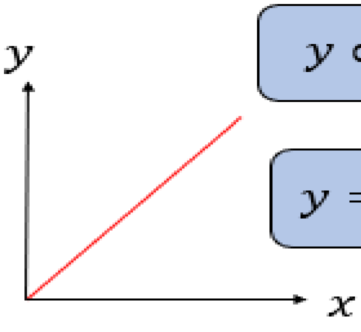
It takes 30 minutes passengers to be checked-in when 10 staff are working. How many staff are needed for 120 minutes?

$$t \propto \frac{1}{s^2}$$
$$t = \frac{k}{s^2}$$
$$30 = \frac{k}{10^2}$$
$$3000 = k$$
$$t = \frac{3000}{s^2}$$

$$t = \frac{3000}{s^2}$$
$$120 = \frac{3000}{s^2}$$
$$s^2 = \frac{3000}{120}$$
$$s^2 = 25$$
$$s = \sqrt{25}$$
$$s = 5$$

Direct and inverse proportion Graphs (H)

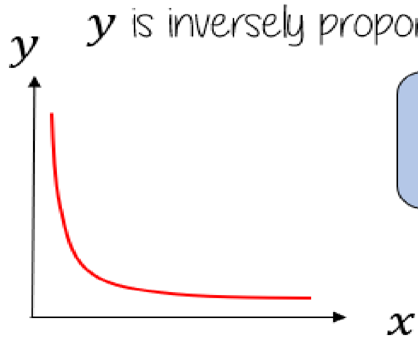
y is directly proportional to x



$$\boxed{y \propto x}$$

$$\boxed{y = kx}$$

y is inversely proportional to x



$$\boxed{y \propto \frac{1}{x}}$$

$$\boxed{y = \frac{k}{x}}$$

YEAR 11F — REASONING…
G17 - GEOMETRIC REASONING

What do I need to be able to do?

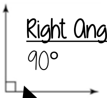
By the end of this unit you should be able to:
- Calculate angles basic angle facts.
- Calculate angles in parallel lines
- Calculate interior and exterior angles
- Prove geometric facts
- Solve problems involving vectors
- Calculate missing lengths in right angle triangles

Keywords

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle
Parallel: Straight lines that never meet
Angle: The figure formed by two straight lines meeting (measured in degrees)
Transversal: A line that cuts across two or more other (normally parallel) lines
Isosceles: Two equal size lines and equal size angles (in a triangle or trapezium)
Polygon: A 2D shape made with straight lines
Regular polygon: All the sides have equal length; all the interior angles have equal size.

Basic angle rules and notation

Acute Angles
0°< angle <90°



Right Angles
90°

Right angle notation

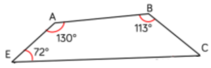
Obtuse
90°< angle <180°

Reflex
180°< angle <360°

Angles on a Straight Line sum to 180°

Reflex
180°< angle <360°

The letter in the middle is the angle
The arc represents the part of the angle



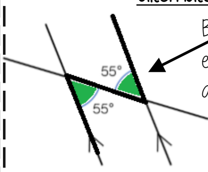
Angle Notation: three letters ABC
This is the angle at B = 113 °

Line Notation: two letters EC
The line that joins E to C.

Vertically opposite angles
Are Equal
Angles around a point sum to 360 °

Angles in parallel lines

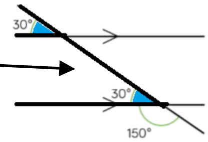
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

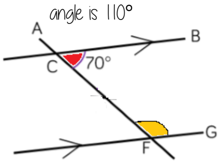
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°

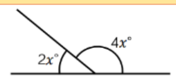


As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Solving angle problems

Link angle facts to algebra	Form an equation	State the reason	Solve

Angles on a straight Line
180°

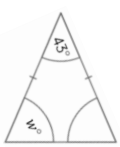


$$2x + 4x = 180°$$

The sum of angles on a straight line is 180 °

$$2x + 4x = 180°$$
$$6x = 180°$$
$$x = 30°$$

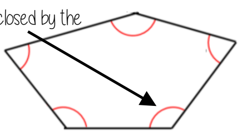
Vertically opposite angles
Equal
Angles around a point
360 °



Triangles
Sum of angles is 180 °

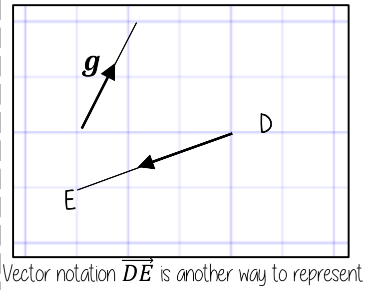
Isosceles have the same base angles

Interior Angles
The angles enclosed by the polygon



(number of sides – 2) x 180

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

The arrow also indicates the direction from point D to point E

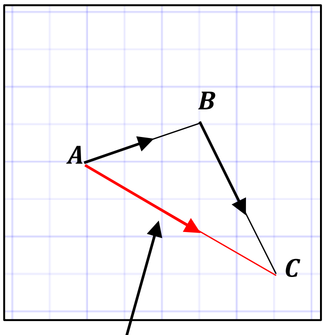
$$\overrightarrow{DE} = \begin{pmatrix}-3\\-1\end{pmatrix}$$

Vectors can also be written in bold lower case so **g** represents the vector

$$\boldsymbol{g} = \begin{pmatrix}1\\2\end{pmatrix}$$

Addition of vectors

$\overrightarrow{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}2\\-4\end{pmatrix}$



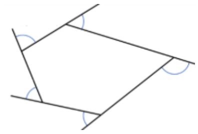
The resultant

$$\overrightarrow{AB} + \overrightarrow{BC}$$
$$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}$$
$$= \begin{pmatrix}3+2\\1+-4\end{pmatrix}$$
$$\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$

Look how this addition compares to the vector $\overrightarrow{AC}$

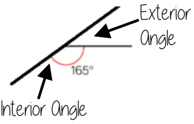
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$$

Sum of exterior angles



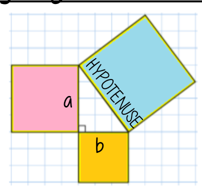
Exterior angles all add up to 360°

Exterior Angles
Are the angle formed from the straight-line extension at the side of the shape



Pythagoras theorem

Hypotenuse^2 = $a^2 + b^2$

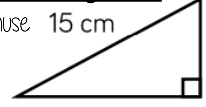


This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Calculate missing sides

Hypotenuse 15 cm



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$
$$144 + b^2 = 225$$
$$b^2 = 111 \quad (-144)$$

Square root to find the length of the side

$$b = \sqrt{111} = 10.54\ cm$$

YEAR 11F — REASONING...
A20 - ALGEBRAIC REASONING

What do I need to be able to do?

By the end of this unit you should be able to:

- Simply complex expressions including a recap on indices
- Find the nth term of a linear sequence
- Solve linear simultaneous equations

Keywords

Arithmetic: a sequence where the difference between the terms is constant

Geometric: a sequence where each term is found by multiplying the previous one by a fixed nonzero number

Sequence: items or numbers put in a pre-decided order

Base: The number that gets multiplied by a power

Power: The exponent — or the number that tells you how many times to use the number in multiplication

Indices: The power or the exponent.

Coefficient: The number used to multiply a variable

Addition/ Subtraction Laws

$a^m \div a^n = a^{m-n}$

$3^5 \div 3^2 = 3^{5-2}$

$= 3^3$

$a^m \times a^n = a^{m+n}$

$3^5 \times 3^2 = 3^{5+2}$

$= 3^7$

The Base number never changes !

Arithmetic/ Geometric sequences

Arithmetic Sequences change by a common difference. This is found by addition or subtraction between terms

Geometric Sequences change by a common ratio. This is found my multiplication/ division between terms.

Term to term rule — how you get from one term (number in the sequence) to the next term.

Position to term rule — take the rule and substitute in a position to find a term. E.g. Multiply the position number by 3 and then add 2

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$

$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

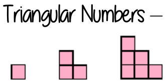
Other sequences

Fibonacci Sequence

1, 1, 2, 3, 5, 8 ...

Each term is the sum of the previous two terms

Triangular Numbers — look at the formation



1, 3, 6, 10, 15 ...

Square Numbers — look at the formation

1, 4, 9, 16...

Sequences are the repetition of a patten

Finding the nth term

This is the 4 times table → 4, 8, 12, 16, 20...

4n

7, 11, 15, 19, 22

This has the same constant difference — but is 3 more than the original sequence

4n + 3

This is the constant difference between the terms in the sequence

This is the comparison (difference) between the original and new sequence

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = \underbrace{2^3 \times 2^3 \times 2^3 \times 2^3}$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12}$$

$a \times b = 3 \times 4 = 12$

NOTICE the difference

$$(2x^3)^4 = \underbrace{2x^3 \times 2x^3 \times 2x^3 \times 2x^3}$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

expressions with indices

$2b^4 \times 3b^2 \equiv 2 \times b \times b \times b \times b \times 3 \times b \times b \rightarrow 6b^6$

$$\frac{5a^3b^2}{15ab^6} \rightarrow \frac{\cancel{5} \times \cancel{a} \times a \times a \times \cancel{b} \times \cancel{b}}{3 \times \cancel{5} \times \cancel{a} \times \cancel{b} \times \cancel{b} \times b \times b \times b \times b} \rightarrow \frac{a^2}{3b^4}$$

There are often misconceptions with this calculation but break down the powers

Simultaneous equations are when more than one equation are given which involve more than one variable. The variables have the same value in each equation.

Solve by addition

Addition makes zero pairs

$3x + 2y = 16$

$+\ 6x - 2y = 2$

$9x = 18$

$\div 9 \qquad \div 9$

$x = 2$

$3x + 2y = 16$

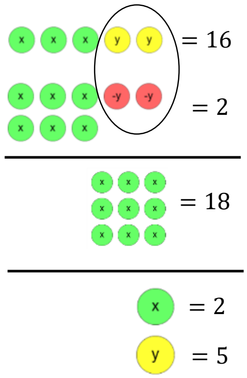
$3(2) + 2(y) = 16$

$6 + 2y = 16$

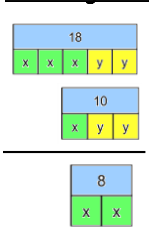
$-6 \qquad -6$

$2y = 10$

$y = 5$



Solve by subtraction



$x = 4$

$y = 3$

$3x + 2y = 18$

$-\ x + 2y = 10$

$2x = 8$

$\div 2 \qquad \div 2$

$x = 4$

$x + 2y = 10$

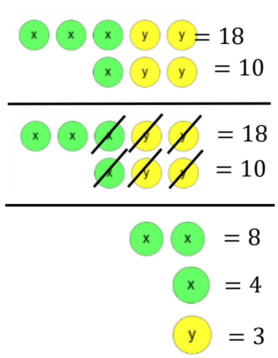
$(4) + 2y = 10$

$-4 \qquad -4$

$2y = 6$

$\div 2 \qquad \div 2$

$y = 3$



Solve the simultaneous equations.

$x5 \quad 2x + 3y = 41$

$x2 \quad 5x + 2y = 42$

$10x + 15y = 205$

$-\ 10x + 4y = 84$

$11y = 121$

$y = 11$

$2x + 3(11) = 41$

$2x + 33 = 41$

$2x = 8$

$x = 4$

Always check your final answer with the other equation to double check your solutions

$y = 11$ and $x = 4$

$5(4) + 2(11) = 42$

$20 + 22 = 42$

$42 = 42$ ✓

YEAR 11F– DELVING INTO DATA…REVISION

COLLECTING, REPRESENTING AND INTERPRETING (1)

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon. two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

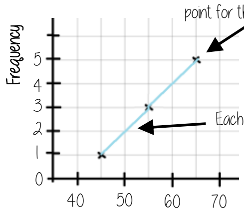
Keywords

Population: the whole group that is being studied
Sample: a selection taken from the population that will let you find out information about the larger group
Representative: a sample group that accurately represents the population
Random sample: a group completely chosen by change. No predictability to who it will include.
Bias: a built-in error that makes all values wrong by a certain amount
Primary data: data collected from an original source for a purpose.
Secondary data: data taken from an external location. Not collected directly.
Outlier: a value that stands apart from the data set

Frequency tables and polygons

x Weight(g)	Frequency
$40 < x \le 50$	1
$50 < x \le 60$	3
$60 < x \le 70$	5

We do not know from grouped data where each value is placed so have to use an estimate for calculations



Each point is plotted at them mid point for the group it represents

Each point is connected with a straight line

The data about weight starts at 40 So the axis can start at 40

MID POINTS
Mid-points are used as estimated values for grouped data. The middle of each group

Mid-point $= \dfrac{\text{Start point + End point}}{2}$

Two way tables

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.

Extract information to input to the two-way table.

Subgroups each have their own heading

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Needs subgroup totals

Overall total

Draw and interpret Pie Charts

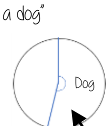
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



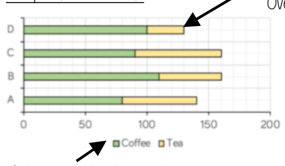
Use a protractor to draw This is 192°

Multiple method
As 60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts:
You NEED the overall frequency to make any comparisons

Bar and line charts

Composite bar charts

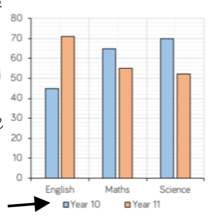


Compare the bars green compared to yellow. The size of each bar is the frequency
Overall total easily comparable

Categories clearly indicated

Dual bar charts

Bars are compared side by side
Easier to compare subgroups



Categories clearly indicated

Averages from a table

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\dfrac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \le 50$	1	45	45
$50 < x \le 60$	3	65	195
$60 < x \le 70$	5	65	325

Overall Frequency: 9

Overall Total : 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

Averages from lists

The Mean

A measure of average to find the central tendency… a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values)
55
Divide the overall total by how many pieces of data you have
$55 \div 5$ Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8

Put the data in order 4, 8, 8, 11, 24
Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data

The modal group – which group has the highest frequency

YEAR 11F— DELVING INTO DATA…REVISION

COLLECTING, REPRESENTING AND INTERPRETING (2)

What do I need to be able to do?

By the end of this unit you should be able to:

- Construct and interpret frequency tables and polygon. two-way tables, line, bar, & pie charts
- Find and interpret averages from a list and a table
- Construct and interpret time series graphs, stem and leaf diagrams and scatter graphs

Keywords

Population: the whole group that is being studied
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Outlier: a value that stands apart from the data set

Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket.

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4 Means 14 years old

Stem and leaf diagrams:
Must include a key to explain what it represents
The information in the diagram should be ordered

Back to back stem and leaf diagrams

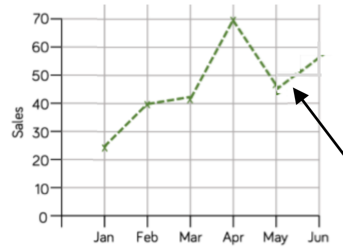
Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

15 | 3,
Means 153 cm tall

Back to back stem and leaf diagrams
Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

Comparing distributions

Comparisons should include a statement of average and central tendency, as well as a statement about spread and consistency

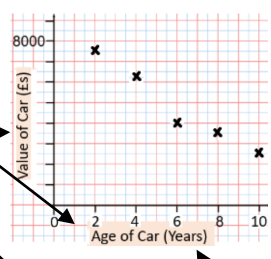
Mean, mode, median – allows for a comparison about more or less average
Range – allows for a comparison about reliability and consistency of data

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled

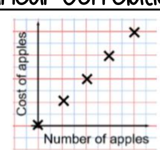


"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

The axis should fit all the values on and be equally spread out

Linear Correlation

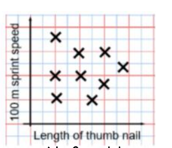


Positive Correlation

As one variable increases so does the other variable

Negative Correlation

As one variable increases the other variable decreases



No Correlation

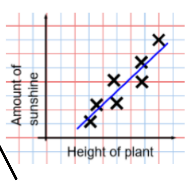
There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

Things to know:

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph



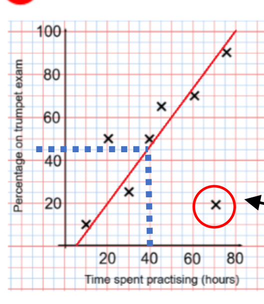
It is only an estimate because the line is designed to be an average representation of the data

It is always a **straight line.**

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data
This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "**outlier**"
It is an outlier because it doesn't fit this model and stands apart from the data

YEAR 11F
KNOWLEDGE ORGANISERS

BLOCK: REVISION AND COMMUNICATION

G18 - TRANSFORMING AND CONSTRUCTING

P5 - LISTING AND DESCRIBING

YEAR 11F — REVISION AND COMMUNICATION...

G18- ROTATION & TRANSLATION (1 OF 3)

What do I need to be able to do?

By the end of this unit you should be able to:

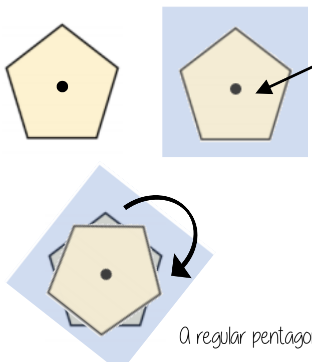
- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement.
Symmetry: when two or more parts are identical after a transformation.
Regular: a regular shape has angles and sides of equal lengths.
Invariant: a point that does not move after a transformation.
Vertex: a point two edges meet.
Horizontal: from side to side
Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

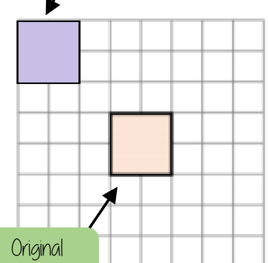
Translation and vector notation

Vector Notation → $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

Translation $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$



Original shape

Every vertex has been translated by the same amount

$\begin{pmatrix} -3 \\ 3 \end{pmatrix}$

The image has been moved 3 squares to the left and 3 squares up

Rotate from a point (in a shape)

Original shape

Point of rotation

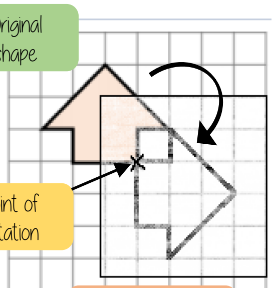


Image: 90° clockwise

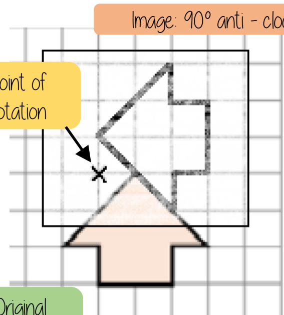
1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

Clockwise   Anti-Clockwise

Rotate from a point (outside a shape)

Image: 90° anti - clockwise

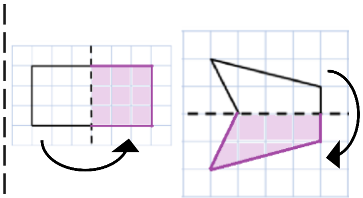
Point of rotation



Original shape

1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

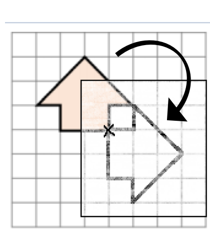
Compare rotations and reflections



Reflections are a mirror image of the original shape.

Information needed to perform a reflection:

- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation

YEAR 11F —REVISION AND COMMUNICATION...
G18 - ENLARGEMENT & SIMILARITY (2 OF 3)

What do I need to be able to do?

By the end of this unit you should be able to:
- Perform and describe transformations
    - Rotation
    - Reflection
    - Translations
    - Enlargements
- Perform standard constructions using a ruler, compass and protractor
- Solve loci problems

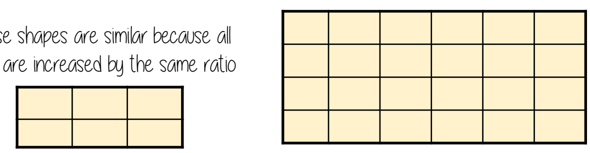
KEYWORDS

Similar Shapes: shapes of different sizes that have corresponding sides in equal proportion and identical corresponding angles.
Scale Factor: the multiple describing how much a shape has been enlarged
Enlarge: to change the size of a shape (enlargement is not always making a shape bigger)
Corresponding: objects (or sides) that appear in the same place in two similar situations.
Image: the picture or visual representation

Recognise enlargement & similarity

Shapes are similar if all pairs of corresponding sides are in the same ratio

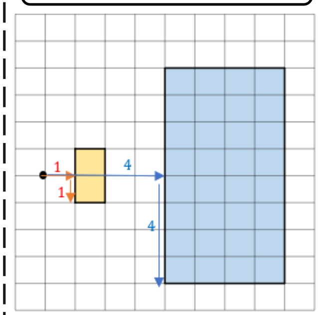
These shapes are similar because all sides are increased by the same ratio



Enlargements are similar shapes with a ratio other than 1

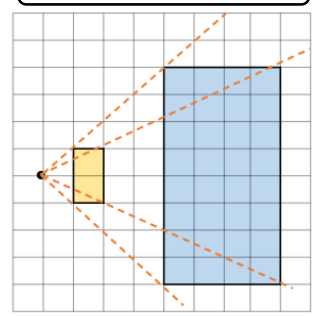
Enlarge a shape from a point

Scaled distances method



Scale the distance between the point of enlargement and each corresponding vertices

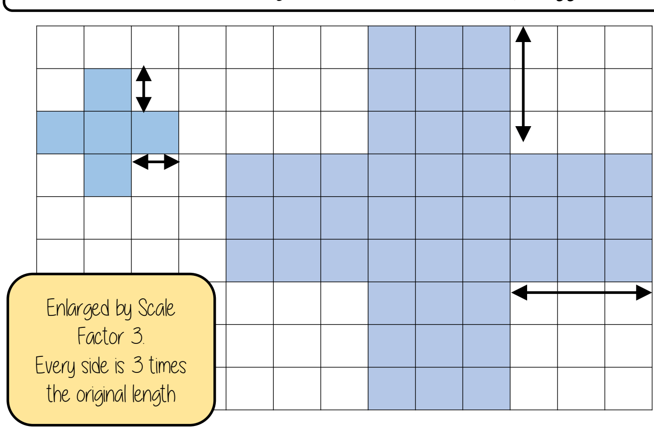
Rays method



Multiply the distance from the centre of corresponding vertices by the scale factor along the ray

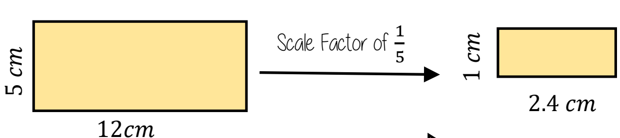
Enlarge by a positive scale factor

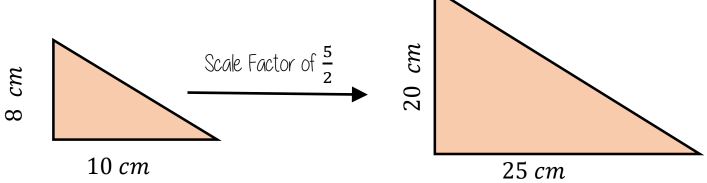
With a scale factor larger than 1 it makes the shape **bigger**



Positive fractional scale factor

With a scale factor between 0 and 1 it makes the shape **smaller**

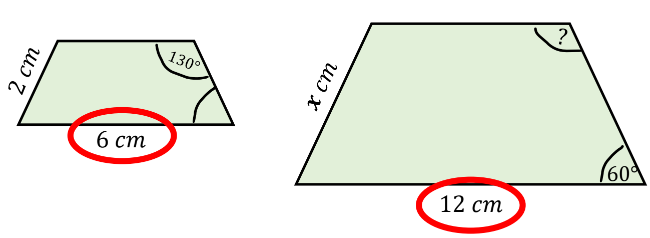




Calculations in similar shapes

Don't forget that properties of shapes don't change with enlargements or in similar shapes

The two trapezium are similar find the missing side and angle



<u>Corresponding sides identify the scale factor</u> $\frac{12}{6} = 2$ Scale Factor = **2**

<u>Calculate the missing side</u> Length (corresponding side) x scale factor

$2cm \times 2$

$x = 4cm$

Enlargement does not change angle size

<u>Calculate the missing angle</u> Corresponding angles remain the same

$130°$

YEAR 11F – REVISION AND COMMUNICATION...
G18 - TRANSFORMING AND CONSTRUCTING (3OF3)

What do I need to be able to do?

By the end of this unit you should be able to:
- Perform and describe transformations
  - Rotation
  - Reflection
  - Translations
  - Enlargements
- Perform standard constructions using a ruler, compass and protractor
- Solve loci problems

Keywords

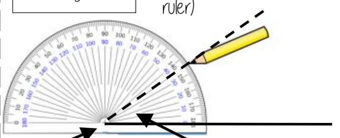
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) – a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

A picture of a car is drawn with a scale of 1:30

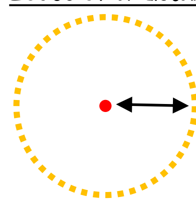
For every 1cm on my image is 30cm in real life

The car image is 10cm

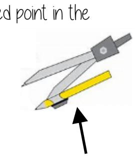
Image : Real life
1cm : 30cm
10cm : 300cm

x 10 x 10

Locus of a distance from a point



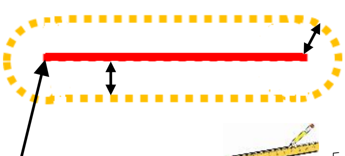
All points are equidistant (the same distance) from the fixed point in the middle.



Equipment needed
The radius is the distance from the fixed point

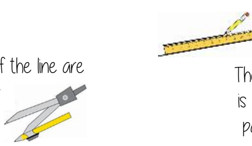
If the point is in the corner it can only make a quarter circle

Locus of a distance from a straight line



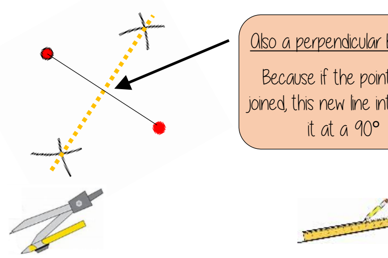
All points are equidistant (the same distance) from line

The ends of the line are fixed points



Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points

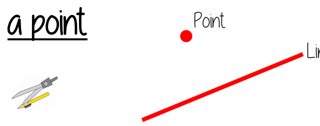


Also a perpendicular bisector
Because if the points are joined, this new line intersects it at a 90°

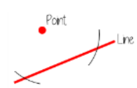
Keep the compass the same size and draw two arcs from each point

Join the intersections with a ruler.
All points on this line are equidistant from both points

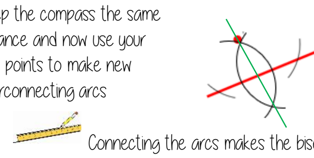
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



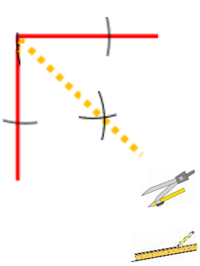
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

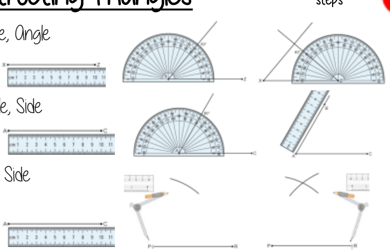
Constructing Triangles

Link to steps R

Side, Angle, Angle

Side, Angle, Side

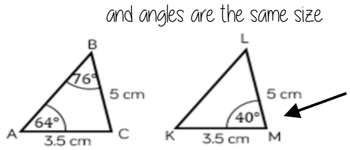
Side, Side, Side



Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

Congruent triangles

Side-side-side
All three sides on the triangle are the same size

Angle-side-angle
Two angles and the side connecting them are equal in two triangles

Side-angle-side
Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side
The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 11F — REVISION AND COMMUNICATION…

P5 - LISTING AND DESCRIBING

What do I need to be able to do?

By the end of this unit you should be able to:

- Work with organised lists
- Work with Sample spaces and probability
- Complete and use Venn diagrams
- Interpret scatter diagrams (info on data knowledge organiser)

Keywords

Event: one or more outcomes from an experiment
Outcome: the result of an experiment.
Intersection: elements (parts) that are common to both sets
Union: the combination of elements in two sets.
Expected Value: the value/ outcome that a prediction would suggest you will get
Universal Set: the set that has all the elements
Systematic: ordering values or outcomes with a strategy and sequence
Product: the answer when two or more values are multiplied together.

Organised lists

Mr and Mrs Lee visit a local restaurant.

They want to choose a starter and then a main meal.

Here are the possible choices.

Starters	Mains
King prawns (K)	Pizza (P)
Soup (S)	Chicken (C)
Halloumi cheese bites (H)	Lasagne (L)
Mushrooms (M)	Fillet steak (F)
	Tomato salad (T)

show the list of possible choices of starter and main.

K, P
K, C
K, L
K, F
K, T

S, P
S, C
S, L
S, F
S, T

H, P
H, C
H, L
H, F
H, T

M, P
M, C
M, L
M, F
M, T

Likeliness of a probability

Impossible
0 or 0%

Even chance
0.5, $\frac{1}{2}$ or 50%

Certain
1 or 100%

The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)

Sum to 1

Probability is always a value between 0 and 1

The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

Experimental data

Theoretical probability	What we expect to happen
Experimental probability	What actually happens when we try it out

The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
Theoretical probability is proportional

Sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

P (Even number and tales) = $\frac{3}{12}$

Independent events

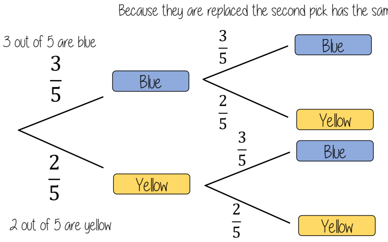
The outcome of two events happening. The outcome of the first event has no bearing on the outcome of the other

P (A and B)
$= P(A) \times P(B)$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick.

Because they are replaced the second pick has the same probability



$= P(BB)$ $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$= P(BY)$ $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

$= P(YB)$ $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

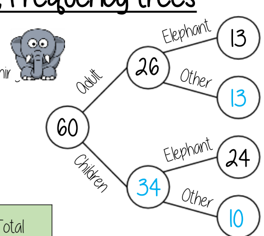
$= P(YY)$ $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

Sum of probabilities = 1

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant.



Frequency trees and two-way tables can show the same information

Two-way table

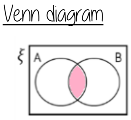
	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

The total columns on two-way tables show the possible denominators

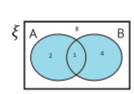
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

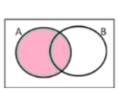
Venn diagram



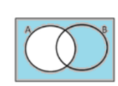
in set A and set B
$P(A \cap B)$



in set A OR set B
$P(A \cup B)$



in set A
$P(A)$



NOT in set A
$P(A')$

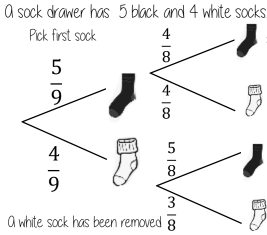
Dependent events

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks, Jamie picks 2 socks from the drawer.

Pick first sock



$= P(BB)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(BW)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(WB)$ $\frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$

$= P(WW)$ $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

Sum of probabilities = 1

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick