Crawshaw Academy

Knowledge Organisers

Year 10 Foundation

A framework for effective home learning

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 1 (Autumn 1) :

A14 - ALGEBRAIC MANIPULATION

A15 - EQUATIONS, INEQUALITIES AND FORMULAE

A16 - QUADRATIC EXPRESSIONS AND EQUATIONS

YEAR 10F — AUTUMN

A14 - ALGEBRAIC MANIPULATION

Sparx Maths
Substitution – U585 (general), U637 (functions), U208 (formulae)
Collect like terms – U105
Simplify expressions – U662 (indices), U105 (like terms), U103 (fractions)
Addition and subtraction laws for indices – U662
Powers of powers – U851
Expand a single bracket – U179 Factorise into a single bracket – U365

What do I need to be able to do?

Step 1 Substitution
Step 2 Collect like terms
Step 3 Simplify expressions
Step 4 Addition and subtraction laws for indices
Step 5 Powers of powers
Step 6 Expand a single bracket
Step 7 Factorise into a single bracket

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: An equation where both sides have variables that cause the same answer includes ≡
Linear: an equation or function that is the equation of a straight line

Like and unlike terms

Like terms are those whose variables are he same

Collecting like terms ≡ symbol

The ≡ symbol means equivalent to.
It is used to identify equivalent expressions

Collecting like terms
Only **like terms** can be combined

$4x + 5b - 2x + 10b$

$4x \quad +5b \quad -2x \quad +10b$

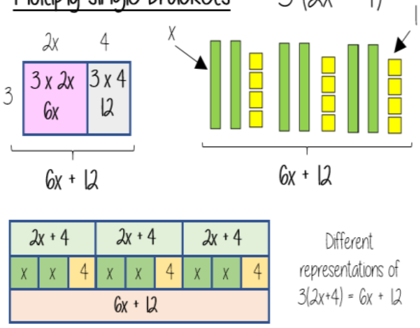
$2x + 15b$

Common misconceptions

$2x + 3x^2 + 4x \equiv 6x + 3x^2$

Although they both have the x variable x2 and x terms are unlike terms so can not be collected

Multiply single brackets $3(2x + 4)$



Different representations of $3(2x+4) = 6x + 12$

Factorise into a single bracket $8x + 4$

$8x + 4$	4
$2x + 1$	

← Try and make this the **highest common factor**

The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
$8x + 4 \equiv 2(4x + 2)$
This is factorised but the HCF has not been used

Algebraic numbers

k is an odd number.

State whether each expression will be odd, even or could be either.

$k - 1$	$2k$	$2k + 1$	$3k$
Even	Even	Odd	Odd

Prove that the sum of two consecutive integers is odd

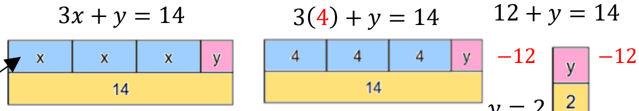
Let n be an integer.
$n + 1$ is 1 greater than n
$n + n + 1 \equiv 2n + 1$
Even

Substituting known variables

A line has the equation $3x + y = 14$

Two different variables, two solutions

Stephanie knows the point x = 4 lies on that line. Find the value for y



$3x + y = 14$

$x = 4$

$3(4) + y = 14$

$12 + y = 14$

$-12 \quad -12$

$y = 2$

Algebraic constructs

Expression A sentence with a minimum of two numbers and one maths operation
Equation A statement that two things are equal
Identity An equation where both sides have variables that cause the same answer includes ≡

Higher powers and roots

x^n ← n – power (number of times multiplied by itself)

x – the base number.

Finding the nth root of any value → $\sqrt[n]{x}$

Other mental strategies for square roots

$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$
$= 9 \times 100$
$= 900$

Addition/ Subtraction Laws

$a^m \div a^n = a^{m-n}$

$a^m \times a^n = a^{m+n}$

Zero and negative indices

$$x^0 = 1$$

Any number divided by itself = 1

$\frac{a^6}{a^6} = a^6 \div a^6$

$= a^{6-6} = a^0 = 1$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$

The same base and power is repeated Use the addition law for indices

$(2^3)^4 = 2^{12}$

NOTICE the difference

$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$(2x^3)^4 = 16x^{12}$

YEAR 10F — AUTUMN

A15 - EQUATIONS, INEQUALITIES AND FORMULAE

Sparx Maths

Solve equations – U325 (two or more steps), U755 (one step)
Solve fractional equations – U505 Solve equations with unknowns on both sides – U870
Understand inequalities – U509
Solve inequalities – U759 (single), U738 (unknowns both sides), U145 (double)
Change the subject of a simple formula – U675 Change the subject of a known formula – U181 Change the subject of a complex formula (E) – U191

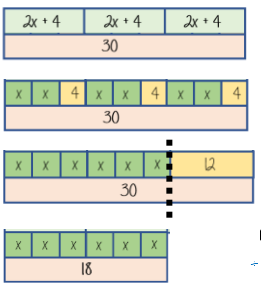
What do I need to be able to do?

- Step 1 Solve equations
- Step 2 Solve fractional equations
- Step 3 Solve equations with unknowns on both sides
- Step 4 Understand inequalities
- Step 5 Solve inequalities
- Step 6 Change the subject of a simple formula
- Step 7 Change the subject of a known formula
- Step 8 Change the subject of a complex formula (E)

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Identity: An equation where both sides have variables that cause the same answer includes ≡
Inequality: an inequality compares two values showing if one is greater than, less than or equal to another.
Fractional Equation – An equation that contains fractions.
Like Terms – Terms that have the same variable and exponent, which can be combined in an equation.
Rearrange – To change the form of an equation or formula to make a different variable the subject.

Solve equations with brackets



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$
$-12 \qquad -12$
$6x = 18$
$\div 6 \qquad \div 6$
$\underline{x = 3}$

Form and solve inequalities

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow \times 3 \leftarrow +2 \leftarrow 11$

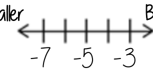
$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$
$+3x \qquad +3x$
$2 > 17 + 3x$
$-17 \qquad -17$
$-15 > 3x$
$\div 3 \qquad \div 3$
$\underline{-5 > x}$

x is true for any value smaller than -5



CHECK IT!
$2 - 3(-6) = 20$
TRUE/ CORRECT

Method 2 Keep the negative x

$2 - 3x > 17$
$-2 \qquad -2$
$-3x > 15$
$\div -3 \qquad \div -3$
$\underline{x > -5}$

x is true for any value bigger than -5

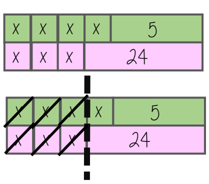
This cannot be true…

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

$4x + 5 = 3x + 24$
$-3x \qquad -3x$
$x + 5 = 24$
$-5 \qquad -5$
$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$
$5x + 20 < 3x + 6$
$2x + 20 < 6$
$2x < -14$
$\underline{x < -7}$

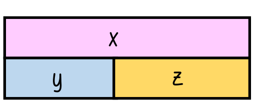
Check it!

$5(-8 + 4) < 3(-8 + 2)$
$5(-4) < 3(-6)$
$-20 < -18$
✓ -20 IS smaller than -18

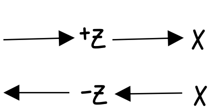
Formulae and Equations

Formulae – all expressed in symbols → **Substitute** in values → **Equations** – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$
Rearrange to make y the subject.
$\underline{y = x - z}$



Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging…

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$
$+3 \qquad +3$
$4x = 12$
$\div 4 \qquad \div 4$
$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$
$+s \qquad +s$
$xy = a + s$
$\div y \qquad \div y$
$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$
Make y the subject first $y = \dfrac{4x + 9}{2}$

YEAR 10F – AUTUMN

A16 - QUADRATIC EXPRESSIONS AND EQUATIONS

Sparx Maths

Expand single brackets and simplify – U179
Factorise into a single bracket – U365 Expand double brackets – U768
Use identities – U582 Factorise quadratic expressions (E) – U178, U858, U963
Solve quadratic equations (E) – U228, U960, U589, U665
Expand triple brackets (E) – U606
Quadratic graphs of the form $y = x^2 + a$ – U989, U667

What do I need to be able to do?

Step 1 Expand double brackets
Step 2 Factorise quadratic expressions (positive only)
Step 3 Factorise quadratic expressions
Step 4 Difference of two squares (E)
Step 5 Solve quadratic equations equal to 0
Step 6 Solve quadratic equations by factorisation
Step 7 Quadratic graphs of the form $y=x^2+a$

Keywords

Simplify: grouping and combining similar terms
Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet.
Equation: an equation says that two things are equal – it will have an equals sign =
Expression: numbers, symbols and operators grouped together to show the value of something
Linear: an equation or function that is the equation of a straight line
Quadratic: a curved graph with the highest power being 2. Square power
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

Expanding double brackets

Double: Where each term in the first bracket is multiplied by all terms in the second bracket. A double bracket will be a quadratic equation.

$(p+2)(2p-1)$ $= 2p^2 + 4p - p - 2$
$= 2p^2 + 3p - 2$

$(p+2)^2 = (p+2)(p+2)$ $= p^2 + 2p + 2p + 4$
$= p^2 + 4p + 4$

Factorising Quadratics

Putting an expression back into brackets. To "factorise fully" means take out the HCF.

Factorise:

$x^2 + 6x + 8$

Add to find the middle term 2+4

Multiply to find the end term: 1 8, ~~2 4~~

$= (x+2)(x+4)$

$x^2 - 2x - 3$

Add to find the middle term −3 + 1

Multiply to find the end term: 1 3

$= (x-3)(x+1)$

Solve when = 0

$-4 + 4 = 0$

Solve the equation $(2x+1)(1-x) = 0$

$(2x+1)(1-x) = 0$

Work with both solution separately

$2x + 1 = 0$
$-1 \quad -1$
$2x = -1$
$\div 2 \quad \div 2$
$x = \frac{-1}{2}$

$1 - x = 0$
$+x \quad +x$
$x = 1$

solve $3x + 4 = 0$
$-4 \quad -4$
So $3x = -4$
$\div 3 \quad \div 3$
$x = \frac{-4}{3}$

Factorise and solve:

$x^2 + 4x - 5 = 0$
$(x-1)(x+5) = 0$

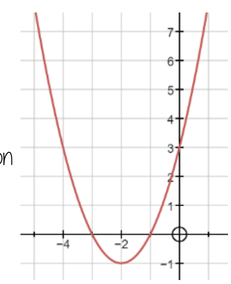
Therefore, the solutions are

Either $x - 1 = 0$
$x = 1$
Or $x + 5 = 0$
$x = -5$

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.



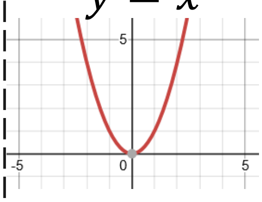
Substitute the x values into the equation of your line to find the y coordinates. Plot all of the coordinate pairs and join the points with a curve (freehand

x	−4	−3	−2	−1	0	1
y	3	0	−1	0	3	8

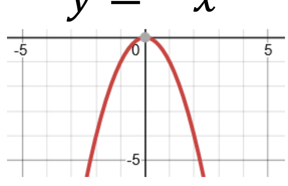
Coordinate pairs for plotting $(-3, 0)$

A quadratic graph will always be in the shape of a parabola.

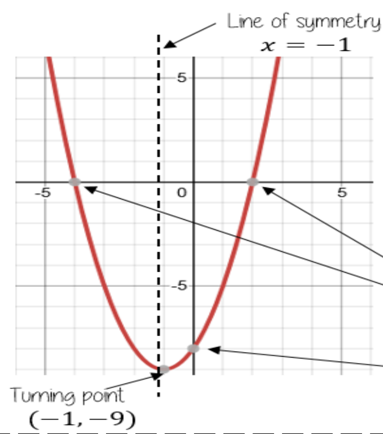
$y = x^2$



$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.



Examples

$y = x^2 + 2x - 8$

A quadratic equation can be solved from its graph. The roots of the graph tell us the possible solutions for the equation. There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the x axis.

Quadratic graphs

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Year 10 HALF TERM 2 (Autumn 2) :

N16 - PERCENTAGES

R6 - RATIO AND SCALE

N17 - WORK WITH FRACTIONS

YEAR 10F — AUTUMN

N16 - PERCENTAGES

Percentage of an amount – U554, U349 Percentage increase and decrease – U773, U671
Repeated percentage change (E) – U332
Express one number as a fraction or a percentage of another – U176, U235
Express a change as a percentage – U278 Find the original value after a percentage change – U286
Simple interest – U533 Compound interest (E) – U332
Choose appropriate methods to solve percentage problems – U278, U554, U286

Sparx Maths

What do I need to be able to do?

- **Step 1** Percentage of an amount
- **Step 2** Percentage increase and decrease
- **Step 3** Repeated percentage change (E)
- **Step 4** Express one number as a fraction or a percentage of another
- **Step 5** Express a change as a percentage
- **Step 6** Find the original value after a percentage change
- **Step 7** Simple interest
- **Step 8** Compound interest (E)
- **Step 9** Choose appropriate methods to solve percentage problems

Keywords

Percent: parts per 100 — written using the % symbol

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.

Fraction: a fraction represents how many parts of a whole value you have.

Equivalent: of equal value.

Reduce: to make smaller in value.

Growth: to increase/ to grow.

Integer: whole number, can be positive, negative or zero.

Invest: use money with the goal of it increasing in value over time (usually in a bank).

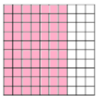
Multiplier: the number you are multiplying by.

Profit: the income take away any expenses/ costs.

Compare FDP

Comparisons are easier in the same format

$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares; 70 "hundredths" = 7 "tenths" 0.7 →



→ 70 hundredths = 70%

Using a calculator: $\frac{■}{□}$ → S⇌D Convert to a decimal

This will give you the answer in the simplest form

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3}$ = 0.3333333 = $0.\dot{3}$
The dot above the 3

Fraction/ Percentage of amount

Find $\frac{3}{5}$ of £60

£60: | £12 | £12 | £12 | £12 | £12 | ; £36

Remember $\frac{3}{5}$ = 60%

10% of £60 = £6
50% of £60 = £30
60% of £60 = £36

Remember $\frac{3}{5}$ = 60% = 0.6
60% of £60
= 0.6 x 60
= £36

Percentage increase/decrease

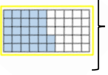
100% ; 42% ; Decrease by 58%

100% - 58% = 42%
1.00 – 0.58 = 0.42 ← Multiplier Less than 1

100% ; 12% ; Increase by 12%

100% + 12% = 112%
1.00 + 0.12 = 1.12 ← Multiplier More than 1

Express as a percentage



27 per every 50 shaded: $\frac{27}{50}$; 54 per every 100 shaded: $\frac{54}{100}$; 54%

$\frac{13}{30}$ → $\frac{■}{□}$ → $\frac{13}{30}$ → × 100 → 43.3333...% → 43%

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage.

Simple and compound interest

Simple Interest

James invests £2000 at 5% simple interest

100% | 5% : £2000 | £100

The original value increases by this amount every year

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100	Original amount: £100
£100 + £10 (10%)	Y1: £110
£110 + £11 (10%)	Y2: £121
£121 + £12.1 (10%)	Y3: £132.10

The multiplier 1.10 repeats each year

Repeated percentage change

Compound Interest

Tess invests £100 at 10% compound interest for 3 years

£100 x 1.10 x 1.10 x 1.10

£100 x 1.10 [3]

Original amount; Repeated multiplier; Number of occurrences

Depreciation

Depreciation calculations use multipliers **less than** 1

Multipliers are commutative – an overall multiplier effect can be calculated by combining the multipliers separately

eg Increase of 10% then a reduction of 10%: x 1.10 x 0.9 → x 0.99 The multiplier

Find the original value

Percentage calculations

Original amount × Multiplier = Final Value

In a test Lucy scored 60% of her questions correctly. Her score was 24. How many questions were on the test.

Original x 0.6 = 24

60%: 24 ; Total questions on test

24 ÷ 0.6 = 40 marks
10% = 6
100% = 40

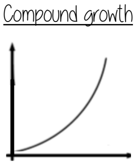
A car sold for a profit £3000 with a profit of 20%. How much was the car originally?

100% | 20% ; £3000

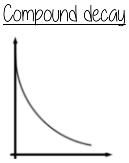
Original x 1.2 = 3000
120% = £3000
10% = £250
100% = £2500

Growth and decay

Compound growth



Compound decay



Compound growth and compound decay are exponential graphs

Decay – the values get closer to 0
The constant multiplier is less than one

Growth – the values increase exponentially
The constant multiplier is more than one

YEAR 10F — AUTUMN

R6 - RATIO AND SCALE

Sparx Maths

Equivalent ratios – M801, M885, U687
Share in a ratio (total given) – M525, U577
Share in a ratio (part or difference given) – M525, U577
Combine a set of ratios (E) – U921
Share in a ratio (algebraically) (E) – U676
Ratios and scales – M112

What do I need to be able to do?

Step 1 Equivalent ratios
Step 2 Share in a ratio (total given)
Step 3 Share in a ratio (part or difference given)
Step 4 Link ratios and fractions
Step 5 Combine a set of ratios (E)
Step 6 Share in a ratio (algebraically) (E)
Step 7 Ratios and scales

Keywords

Ratio: a statement of how two numbers compare
Equivalent: of equal value
Proportion: a statement that links two ratios
Integer: whole number, can be positive, negative or zero.
Fraction: represents how many parts of a whole.
Denominator: the number below the line on a fraction. The number represent the total number of parts.
Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Origin: (0,0) on a graph. The point the two axes cross
Gradient: The steepness of a line

Compare with ratio

"For every dog there are 2 cats"

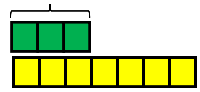
Dogs: Cats
1:2

The ratio has to be written in the same order as the information is given.
e.g. 2:1 would represent 2 dogs for every 1 cat.

Units have the be of the same value to compare ratios

Ratios and fraction

Trees: Flowers
3 : 7



Ratio

Fraction of trees

Number of parts of in group / Total number of parts → $\frac{3}{10}$

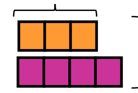
Fraction

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

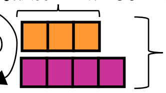
Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50
□ = one part = £50

Put back into the question

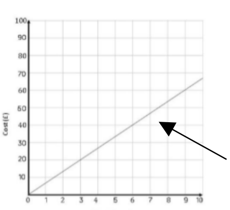
James: Lucy
x 50 3 : 4 x 50
£150:£200

James = 3 x £50 = £150



Lucy = 4 x £50 = £200

Ratio and graphs



Graphs with a constant ratio are directly proportional
- Form a straight line
- Pass through (0,0)

The gradient is the constant ratio

Ratio and scale

A picture of a car is drawn with a scale of 1:30

The car image is 10cm

Image : Real life
x 10 1cm : 30cm x 10
10cm : 300cm

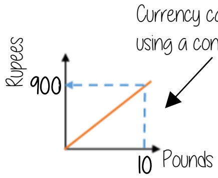
Conversion between currencies

£1 = 90 Rupees ← Currency is directly proportional

For every £1 I have 90 Rupees

x 10 £1 = 90 Rupees x 10
£10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

x 7 £1 = 90 Rupees x 7 ← 630 ÷ 90 = 7
£7 = 630 Rupees

Ratios in 1:n and n:1

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of 1:n

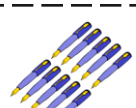
The question states that **this part has to be 1 unit**. Therefore **Divide by 4**

4 : 20
1 : 5

This side has to be divided by 4 too – to keep in **proportion**

the n part does not have to be an integer for this type of question

Best buys



	4 pens costs £2.60	10 pens costs £6.00
"1 pen costs…"	£2.60 ÷ 4 = £0.65	£6.00 ÷ 10 = £0.60
"1 pound buys…"	4 ÷ 2.60 = 1.54 pens	10 ÷ 6 = 1.67 pens

You could work out how much 40 pens are and then compare

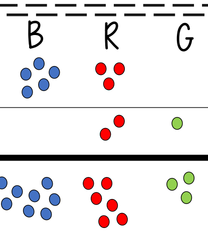
Compare the solution in the context of the question

The best value has the lowest cost "per pen"

The best value means £1 buys you more pens

Combining ratios

The ratio of Blue counters to Red counters is 5:3
The ratio of Red counters to Green counters is 2:1
Ratio of Blue to Red to Green



10 : 6 : 3

Use equivalent ratios to allow comparison of the group that is common to both statements

Lowest common multiple of the ratio both statements share

YEAR 10F — AUTUMN

N17 - WORK WITH FRACTIONS

Sparx Maths

Fraction of an amount – U881, U916 Increase or decrease an amount by a fraction – U475, U736
Use a fraction to find the whole – U916 (reverse operations implied)
Equivalent fractions and mixed numbers – U704, U692 Add and subtract fractions – U736
Multiply fractions – U475 Divide fractions – U544
Solve problems with fractions – U736, U881, U916

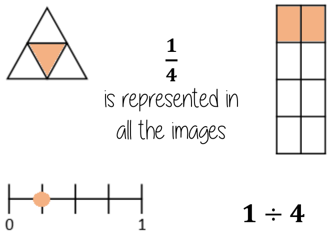
What do I need to be able to do?

Step 1 Fraction of an amount
Step 2 Increase or decrease an amount by a fraction
Step 3 Use a fraction to find the whole
Step 4 Equivalent fractions and mixed numbers
Step 5 Add and subtract fractions
Step 6 Multiply fractions
Step 7 Divide fractions
Step 8 Solve problems with fractions

Keywords

Numerator: the number above the line on a fraction. The top number. Represents how many parts are taken
Denominator: the number below the line on a fraction. The number represent the total number of parts
Equivalent: of equal value
Mixed numbers: a number with an integer and a proper fraction
Improper fractions: a fraction with a bigger numerator than denominator
Substitute: replace a variable with a numerical value
Place value: the value of a digit depending on its place in a number. In our decimal number system, each place is 10 times bigger than the place to its right

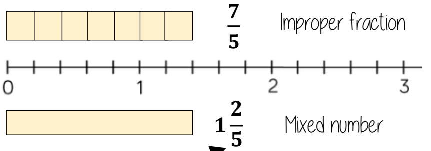
Representing Fractions



$\frac{1}{4}$ is represented in all the images

$1 \div 4$

Mixed numbers and fractions



$\frac{7}{5}$ Improper fraction

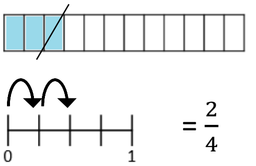
$1\frac{2}{5}$ Mixed number

In this model 5 parts make up a whole

Fractions can be bigger than a whole

Add/Subtract unit fractions

Same denominator



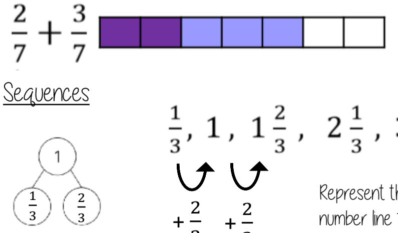
$\frac{1}{12} + \frac{1}{12} - \frac{1}{12} = \frac{2}{12}$

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

With the same denominator ONLY the numerator is added or subtracted

Add/Subtract fractions

Same denominator



$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

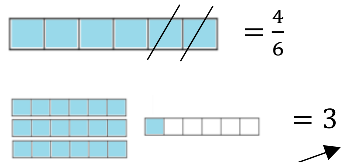
Sequences

$\frac{1}{3}, 1, 1\frac{2}{3}, 2\frac{1}{3}, 3, \ldots$

$+\frac{2}{3}$ $+\frac{2}{3}$

Represent this on a number line to help

Add/Subtract from integers



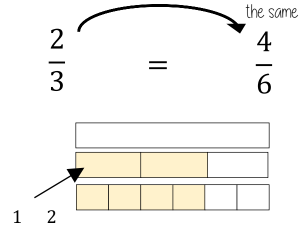
$1 - \frac{2}{6} = \frac{4}{6}$

$3 + \frac{1}{6} = 3\frac{1}{6}$

The denominator indicates the number of parts a whole is made up of

Equivalent fractions

Numerator and denominator have the same multiplier



$\frac{2}{3} = \frac{4}{6}$

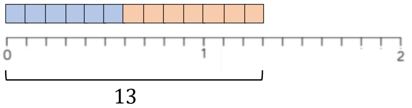
$\frac{1}{3} = \frac{2}{6}$

Add/Subtraction fractions (common multiples)

$\frac{3}{5} + \frac{7}{10}$

$\frac{6}{10} + \frac{7}{10}$

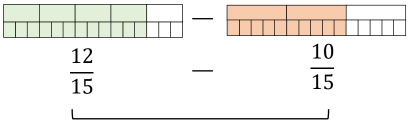
Addition/Subtraction needs a common denominator



$\frac{13}{10}$

Add/Subtraction any fractions

$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$

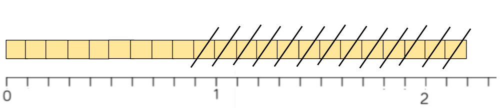
Use equivalent fractions to find a common multiple for both denominators

Add/Subtraction fractions (improper and mixed)

$2\frac{1}{5} - 1\frac{3}{10}$

$2\frac{2}{10}$

$\frac{22}{10} - \frac{13}{10} = \frac{9}{10}$



- Convert to an improper fraction
- Calculate with common denominator

Partitioning method

$2\frac{1}{5} - 1\frac{3}{10} = 2\frac{2}{10} - 1\frac{3}{10} = 2\frac{2}{10} - 1 - \frac{3}{10} = 1\frac{2}{10} - \frac{3}{10} = \frac{9}{10}$

Quick Multiplying and Cancelling down

The 3 and the 9 have a common factor and can be simplified

$\frac{\cancel{3}^{1}}{5} \times \frac{4}{\cancel{9}^{3}} = \frac{1 \times 4}{5 \times 3} = \frac{4}{15}$

Quick Solving
Multiply the numerators
Multiply the denominators

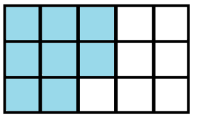
Dividing any fractions *Remember to use reciprocals*

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

Represented



$= \frac{8}{15}$

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 3 (SPRING 1) :

N18 - NON-CALCULATOR METHODS

A17 - STRAIGHT LINE GRAPHS

P3 – PROBABILITY

N19 - ROUNDING AND ESTIMATING

YEAR 10F — SPRING

N18 - NON-CALCULATOR METHODS

Sparx Maths

Place value for integers and decimals – M522, M704, M911
Compare and order numbers – M335, M521, M527
Add and subtract integers and decimals – M106, M152, M336
Multiply and divide integers and decimals – M113, M157, M187
Four operations with directed number – M106, M288, M527
Order of operations – M521 Related calculations – M409, M952

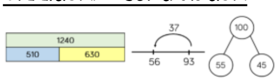
What do I need to be able to do?

Step 1 Place value for integers and decimals
Step 2 Compare and order numbers
Step 3 Add and subtract integers and decimals
Step 4 Multiply and divide integers and decimals
Step 5 Four operations with directed number
Step 6 Order of operations
Step 7 Related calculations
Step 8 Solve multi-step problems

Keywords

Truncate: to shorten, to shorten a number (no rounding), to shorten a shape (remove a part of the shape)
Round: making a number simpler, but keeping its place value close the what it originally was
Credit: money that goes into a bank account **Debit**: money that leaves a bank account
Profit: the amount of money after income - costs
Tax: money that the government collects based on income, sales and other activities.
Overestimate: Rounding up – gives a solution higher than the actual value
Underestimate: Rounding down – gives a solution lower than the actual value
Integer Whole numbers; can be positive or negative
Rational Numbers All integers plus fractions (they are made by diving 2 integers
Irrational numbers Numbers that cannot be written as a fractions e.g. π and $\sqrt{2}$

Addition/ Subtraction



Modelling methods for addition/ subtraction
- Bar models
- Number lines
- Part/ Whole diagrams

Addition is commutative

6 + 3 = 3 + 6

The order of addition does not change the result

Subtraction the order has to stay the same

360 – 147 = 360 – 100 – 40 – 7

- Number lines help for addition and subtraction
- Working in 10's first aids mental addition/ subtraction
- Show your relationships by writing fact families

Formal written methods

	H	T	O
	1	8	7
+	5	4	2

	H	T	O
	4	2	7
–	2	4	9

Remember the place value of each column. You may need to move 10 ones to the ones column to be able to subtract

Decimals have the same methods remember to align the place value

Division methods

3584 ÷ 7 = 512

Short division

$$7\overline{)3\,{}^{3}5\,8\,{}^{1}4} = 5\;1\;2$$

Complex division

÷ 24 = ÷ 6 ÷ 4

Break up the divisor using factors

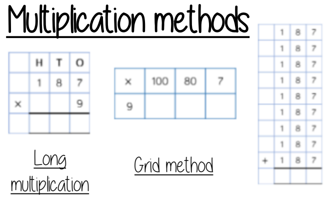
Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

24 ÷ 0.02 ⟶ 24 ÷ 0.2 ⟶ 240 ÷ 2

All give the same solution as represent the same proportion
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods



Start with the representation of 2
Less effective method especially for bigger multiplication

Multiplication with decimals

Perform multiplications as integers
e.g. 0.2 x 0.3 ⟶ 2 x 3

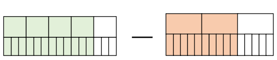
Make **adjustments** to your answer to match the question: 0.2 x 10 = 2
0.3 x 10 = 3

Therefore 6 ÷ 100 = 0.06

Four operations with fractions

Addition and Subtraction

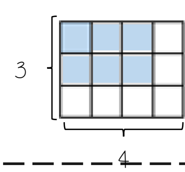
$\frac{4}{5} - \frac{2}{3}$



$\frac{12}{15}$ $\frac{10}{15}$ $= \frac{2}{15}$

Multiplication

$\frac{3}{4} \times \frac{2}{3}$



Division

$\frac{2}{5} \div \frac{3}{4}$

$\frac{2}{5} \times \frac{4}{3}$

Multiplying by a reciprocal gives the same outcome

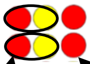
$= \frac{8}{15}$

Directed number

● = –1
● = 1

Addition

2 + - 4 = -2

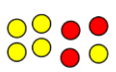


Generalisation
+ – = –

Zero pair (-1 + 1 = 0)

Two " – 1 " left = -2

Subtraction



= 2

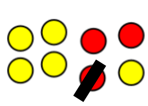
Representation for calculation

Generalisation
– – = +

"Subtract" – means take away or remove

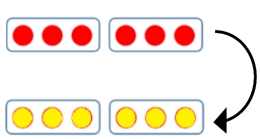
2 - - 1 = 3

Take away one ●



= 3

Multiplication



The act of making counters into their negative is turning them over

-2 x -3 = 6

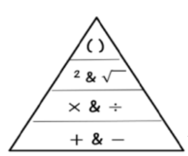
Divisions are the inverse operations

a = 5   b = -4

Brackets around negative substitutions helps remove calculation errors

2a – b = 2 x 5 – (-4) = 10 + 4 = 14

Use order or operations



Brackets
Indices or roots
Multiplication or division
Addition or subtraction

Brackets around negative substitutions helps remove calculation errors

Remember square roots have a positive and negative value

×	–3	–2	–1	0	1	2	3
–3	9	6	3	0	–3	–6	–9
–2	6	4	2	0	–2	–4	–6
–1	3	2	1	0	–1	–2	–3
0	0	0	0	0	0	0	0
1	–3	–2	–1	0	1	2	3
2	–6	–4	–2	0	2	4	6
3	–9	–6	–3	0	3	6	9

YEAR 10F – SPRING
A17 - STRAIGHT LINE GRAPHS

Plot straight line graphs – M932, U741 Find solutions to equations using straight line graphs – U741
Explore gradients – U477, U800 y = mx + c – M544, U315, U669
Find the equation of a line from a graph (E) – U477, U848
Find the midpoint of a line segment – M622, M311
Equation of a straight-line graph given one point and a gradient – U477
Equation of a straight-line graph given two points (E) – U848 Real-life straight-line graphs (E) – U652, U862

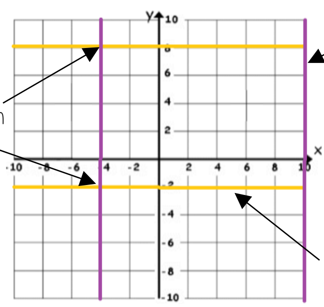
What do I need to be able to do?

Step 1 Plot straight line graphs
Step 2 Find solutions to equations using straight line graphs
Step 3 Explore gradients
Step 4 y = mx + c
Step 5 Find the equation of a line from a graph (E)
Step 6 Find the midpoint of a line segment
Step 7 Equation of a straight-line graph given one point and a gradient
Step 8 Equation of a straight-line graph given two points (E)
Step 9 Real-life straight-line graphs (E)

Keywords

Gradient: the steepness of a line
Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.
Parallel: two lines that never meet with the same gradient.
Co-ordinate: a set of values that show an exact position on a graph.
Linear: linear graphs (straight line) – linear common difference by addition/ subtraction
Asymptote: a straight line that a graph will never meet.
Reciprocal: a pair of numbers that multiply together to give 1.
Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes



All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Intersection points

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

e.g (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

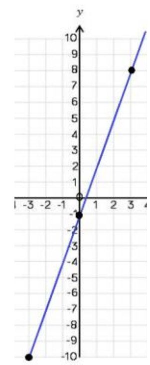
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

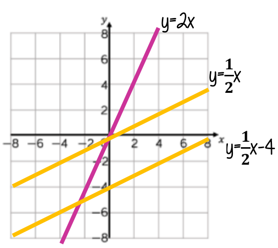
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$$y = mx + c$$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The **greater** the gradient – the **steeper** the line

Parallel lines have the **same** gradient

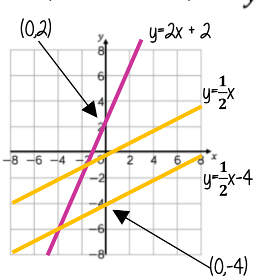
Positive gradients

Negative gradients

Compare Intercepts

$$y = mx + c$$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$$y = mx + c$$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**.

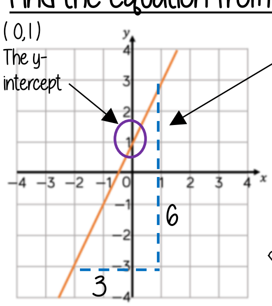
The equation of a line can be rearranged: E.g

y = c + mx

c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$$y = 2x + 1$$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge. The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost. The gradient shows the price per pen

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

YEAR 10F — SPRING

P3 — PROBABILITY

Sparx Maths

Find the probability of a single event — U510
Use the property that probabilities sum to 1 — U510 List outcomes — U104
Relative frequency — U580 Sample spaces for 1 or more events — U104
Two-way tables — U981 Frequency trees — U280 Independent events — U558
Tree diagrams for independent events — U558
Tree diagrams for dependent events (E) — U729

What do I need to be able to do?

- Step 1 Find the probability of a single event
- Step 2 Use the property that probabilities sum to 1
- Step 3 List outcomes
- Step 4 Relative frequency
- Step 5 Sample spaces for 1 or more events
- Step 6 Two-way tables
- Step 7 Frequency trees
- Step 8 Independent events
- Step 9 Tree diagrams for independent events
- Step 10 Tree diagrams for dependent events (E)

Keywords

Event: one or more outcomes from an experiment
Outcome: the result of an experiment.
Intersection: elements (parts) that are common to both sets
Union: the combination of elements in two sets.
Expected Value: the value/ outcome that a prediction would suggest you will get
Universal Set: the set that has all the elements
Systematic: ordering values or outcomes with a strategy and sequence
Product: the answer when two or more values are multiplied together.

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times

$0.3 \times 100 = 30$

Single event probability

Probability is always a value between 0 and 1

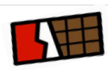
The probability of getting a blue ball is $\frac{1}{5}$
∴ The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is **1**

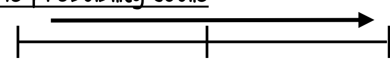
The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

P(white chocolate) = 1 – 0.15 – 0.35
= 0.5



The probability scale

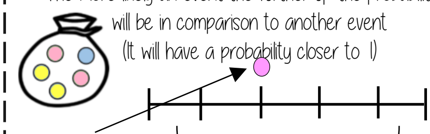


Impossible
0 or 0%

Even chance
0.5, $\frac{1}{2}$ or 50%

Certain
1 or 100%

The more likely an event the further up the probability it will be in comparison to another event
(It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes
So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Experimental data

Theoretical probability — What we expect to happen

Experimental probability — What actually happens when we try it out

The more trials that are completed the closer experimental probability and theoretical probability become

The probability becomes more accurate with more trials.
Theoretical probability is proportional

Sample space

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

P (Even number and tales) = $\frac{3}{12}$

Independent events

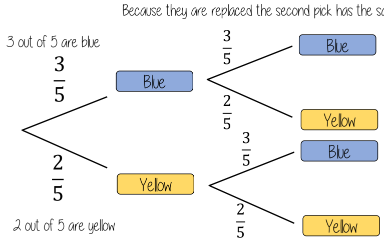
The outcome of two events happening The outcome of the first event has no bearing on the outcome of the other

$P(A \text{ and } B)$
$= P(A) \times P(B)$

Tree diagram for independent event

Isobel has a bag with 3 blue counters and 2 yellow. She picks a counter and replaces it before the second pick

Because they are replaced the second pick has the same probability



$= P(BB)$ $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

$= P(BY)$ $\frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$

$= P(YB)$ $\frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

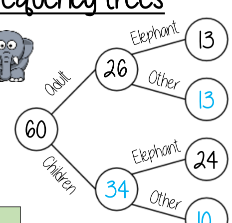
$= P(YY)$ $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$

Sum of probabilities = 1

Tables, Venn diagrams, Frequency trees

Frequency trees

60 people visited the zoo one Saturday morning. 26 of them were adults. 13 of the adult's favourite animal was an elephant. 24 of the children's favourite animal was an elephant



Frequency trees and two-way tables can show the same information

The total columns on two-way tables show the possible denominators

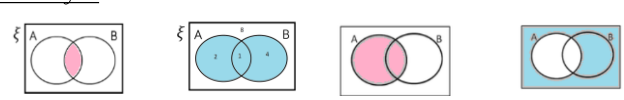
P(adult) = $\frac{26}{60}$

P(Child with favourite animal as elephant) = $\frac{13}{37}$

Two-way table

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Venn diagram



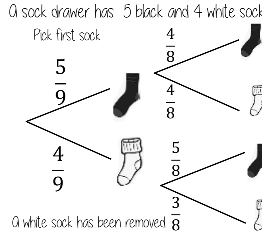
in set A AND set B	in set A OR set B	in set A	NOT in set A
$P(A \cap B)$	$P(A \cup B)$	$P(A)$	$P(A')$

Dependent events

The outcome of the first event has an impact on the second event

Tree diagram for dependent event

A sock drawer has 5 black and 4 white socks, Jamie picks 2 socks from the drawer.



$= P(BB)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(BW)$ $\frac{5}{9} \times \frac{4}{8} = \frac{20}{72}$

$= P(WB)$ $\frac{4}{9} \times \frac{5}{8} = \frac{20}{72}$

$= P(WW)$ $\frac{4}{9} \times \frac{3}{8} = \frac{12}{72}$

Sum of probabilities = 1

NOTE: as "socks" are removed from the drawer the number of items in that drawer is also reduced ∴ the denominator is also reduced for the second pick

YEAR 10F – SPRING

N19 - ROUNDING AND ESTIMATING

Sparx Maths

Round to decimal places – M431
Round to significant figures – M994 (integers), M131 (decimals)
Estimate answers to calculations – M878
Use of a calculator – U161 (standard form), U349 (percentages), U916 (fractions)
Error intervals (including truncation) (E) – U657, U301, U108

What do I need to be able to do?

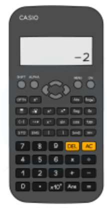
Step 1 Round to decimal places
Step 2 Round to significant figures
Step 3 Estimate answers to calculations
Step 4 Use of a calculator
Step 5 Error intervals (including truncation) (E)

Keywords

Significant figure – The digits in a number that carry meaning contributing to its precision (starting from the first non-zero digit).
Rounding – Reducing the digits in a number while keeping its value close to the original
Approximation – A value or quantity that is nearly but not exactly correct.
Estimate – A rough calculation of the value, number, or quantity.
Error interval – A range within which a number lies after rounding.
Upper bound – The highest possible value in an error interval
Lower bound – The lowest possible value in an error interval
Accuracy – How close a measured or calculated value is to the true value.

Using a Calculator

Check Mode – Make sure it's in **Degrees** and **Maths/Comp** mode.
Brackets – Use them to keep the correct order of operations.
Negatives – Use the (–) key for negative numbers.
Squares/Roots – Use X^2 for squares and $\sqrt{x}$ for roots.
Fractions – Use the fraction button to enter and simplify.
Standard Form – Use the EXP or $x10^x$ button.
ANS – Reuse your last answer with the ANS key.



Rounding

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47

2.46 | 247

2.46|192 — This shows the number is closer to 2.46

Significant Figures
3.70 to 1 significant figure is 400
37 to 1 significant figure is 40
3.7 to 1 significant figure is 4
0.37 to 1 significant figure is 0.4
0.00000037 to 1 significant figure is 0.0000004

SF: Round to the first nonzero number

Round to decimal places

Focus on the numbers **after** the decimal point

"To 1dp" – to one number after the decimal
"To 2dp" – to two numbers after the decimal

2.46192 (to 1dp) - Is this closer to 2.4 or 2.5

2.4 | 2.5

2.4|6192 — This shows the number is closer to 2.5

2.46192 (to 2dp) - Is this closer to 2.46 or 2.47

2.46 | 2.47

2.46|192 — This shows the number is closer to 2.46

Estimate the calculation

Round to 1 significant figure to estimate

4.2 + 6.7 ≈ 4 + 7 ≈ 11

This is an **overestimate** because the 6.7 was rounded up more

The equal sign changes to show it is an estimation

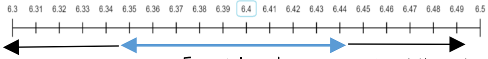
21.4 x 3.1 ≈ 20 x 3 ≈ 60

This is an **underestimate** because both values were rounded down

It is good to check all calculations with an estimate in all aspects of maths – it helps you identify calculation errors.

Limits of accuracy

A width w has been rounded to 6.4cm correct to 1dp.



< 6.35 the values would round to 6.3 | Error interval | ≥ 6.45 the values would round to 6.5

The error interval: $6.35 \le w < 6.45$

Any value within these limits would round to 6.4 to 1dp

A width w has been truncated to 6.4cm correct to 1dp.

< 6.4 the values would truncate to 6.3 | Error interval | ≥ 6.5 the values would truncate to 6.5

$6.4 \le w < 6.5$

Any value within these limits would truncate to 6.4 to 1dp

Solve problems with estimation

Estimating a Sum

Question: Estimate the total cost of items costing £9.85, £3.20, and £7.60.

Round £9.85 → £10
Round £3.20 → £3
Round £7.60 → £8
Estimated total = 10 + 3 + 8 = **£21**

Estimating Per Person Cost

Question: A group meal costs £187.65, shared between 9 people. Estimate the cost per person.

Solution:
Round £187.65 → £200
Round 9 → 10 (9 is close to 10, so it's a good enough estimate, although it should be 1sf)
200÷10=**£20**

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 4 (Spring 2):

G13 - PERIMETER, AREA AND VOLUME

S5 - INTERPRET AND REPRESENT DATA

A18 - NON-LINEAR GRAPHS

YEAR 10F – SPRING

G13 - PERIMETER, AREA AND VOLUME

Sparx Maths

Name 2-D and 3-D shapes – M276 (2D), M767 (3D) Perimeter of a 2-D shape – M635, M690 Area of a 2-D shape – M390 (rectangles), M610 (triangles), M291 (parallelograms), M705 (trapeziums) Area of a compound shape – M269, M996 Recognise and label parts of a circle – M595 Circumference of a circle – M169 Area of a circle – M231 Volume of a prism – M722 Nets – M518 Surface area of a prism – M661

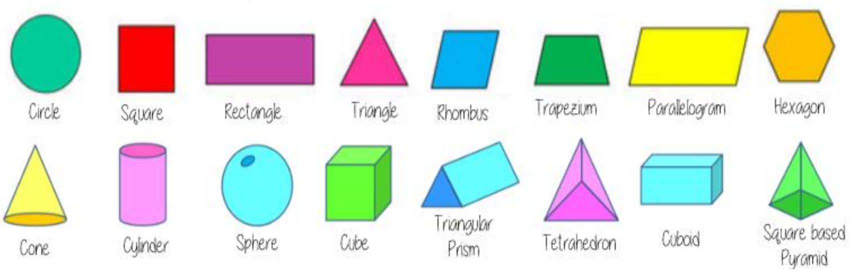
What do I need to be able to do?

Step 1 Name 2-D and 3-D shapes
Step 2 Perimeter of a 2-D shape
Step 3 Area of a 2-D shape
Step 4 Area of a compound shape
Step 5 Recognise and label parts of a circle
Step 6 Circumference of a circle
Step 7 Area of a circle
Step 8 Volume of a prism
Step 9 Nets
Step 10 Surface area of a prism

Keywords

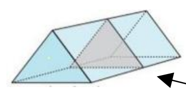
2D: two dimensions to the shape e.g. length and width
3D: three dimensions to the shape e.g. length, width and height
Vertex: a point where two or more lines segments meet
Edge: a line on the boundary joining two vertex
Face: a flat surface on a solid object
Cross-section: a view inside a solid shape made by cutting through it
Plan: a drawing of something when drawn from above (sometimes birds eye view)
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

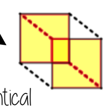
Name 2D & 3D shapes



Recognise prisms

A solid object with two identical ends and flat sides

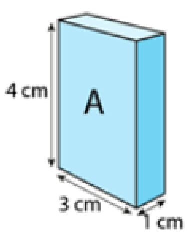


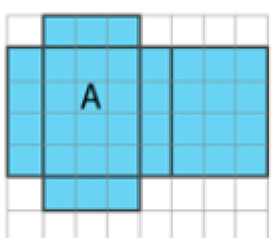


The cross section will also be identical to the end faces.

A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

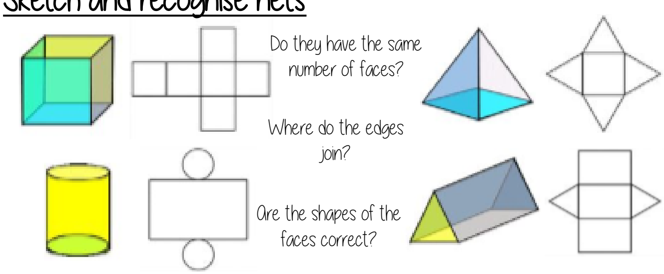




1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

Sketch and recognise nets

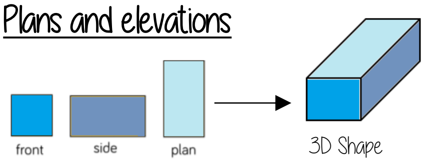


Do they have the same number of faces?

Where do the edges join?

Are the shapes of the faces correct?

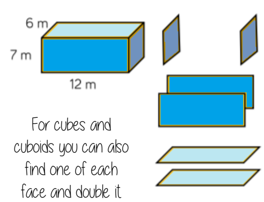
Plans and elevations



The direction you are considering the shape from determines the front and side views

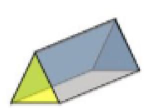
Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



Sides	6 x 7	
	6 x 7	
Front and back	12 x 7	
	12 x 7	
Top and Bottom	12 x 6	
	12 x 6	Sum of all sides is surface area

For cubes and cuboids you can also find one of each face and double it



For other shapes – not all the sides are the same, so calculate the individually

Volumes

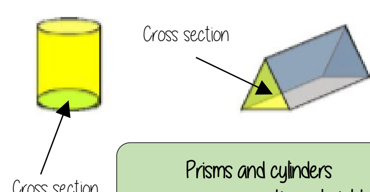
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

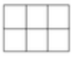
Remember multiplication is commutative



Prisms and cylinders = area cross section x height

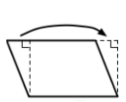
Height can also be described as depth

Area of 2D shapes



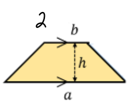
Rectangle
Base x Height

Triangle
½ x Base x Perpendicular height

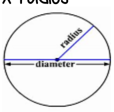


Parallelogram/ Rhombus
Base x Perpendicular height

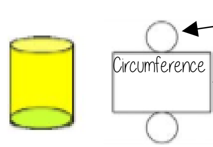
Area of a trapezium
$\frac{(a+b) \times h}{2}$



Area of a circle
π x radius²



Surface area - cylinders



The area of the circle
π x radius²

The width of this face is the same as the circumference
π x diameter x height

2 x π x radius² + π x diameter x height

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 10F — SPRING

S5 - INTERPRET AND REPRESENT DATA

Sparx Maths

Averages and range – U526, U456, U260, U291 Averages from an ungrouped frequency table – U569 Mean from a grouped frequency table – U877 Averages from a grouped frequency table – U877 Use data to compare distributions (E) – U507 Types of data – U322 Sampling – U162 Scatter graphs – U199, U277 Interpolation and extrapolation (E) – U277

What do I need to be able to do?

- **Step** 1 Averages and range
- **Step** 2 Averages from an ungrouped frequency table
- **Step** 3 Mean from a grouped frequency table
- **Step** 4 Averages from a grouped frequency table
- **Step** 5 Use data to compare distributions (E)
- **Step** 6 Types of data
- **Step** 7 Sampling
- **Step** 8 Scatter graphs
- **Step** 9 Interpolation and extrapolation (E)

Keywords

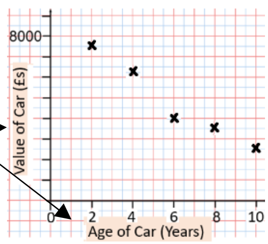
Data – Information collected for analysis; can be qualitative (words) or quantitative (numbers).
Outlier – A value that is much higher or lower than the rest of the data
Error – A mistake in data collection or recording
Mean – The average, found by adding all values and dividing by the number of values.
Median – The middle value when data is in order.
Mode – The most frequent value in a data set.
Range – The difference between the highest and lowest values.
Frequency Table – A table showing how often each value or group of values occurs.
Grouped Data – Data that is organized into intervals or classes.
Distribution – The way data is spread out, often compared using averages and range.

Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£s)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship

All axes should be labelled

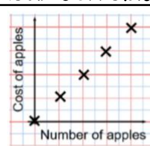


"This scatter graph show as the age of a car increases the value decreases"

The link between the data can be explained verbally

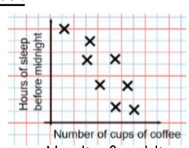
The axis should fit all the values on and be equally spread out

Linear Correlation



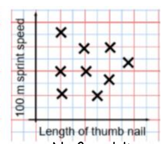
Positive Correlation

As one variable increases so does the other variable



Negative Correlation

As one variable increases the other variable decreases

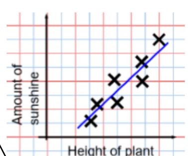


No Correlation

There is no relationship between the two variables

The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph



Things to know:

- The line of best fit DOES NOT need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line (It may not go through any points)
- The line extends across the whole graph

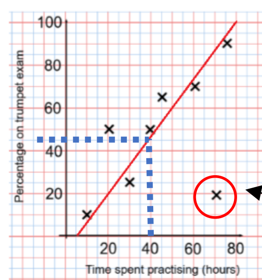
It is only an estimate because the line is designed to be an average representation of the data

It is always a straight line.

Using a line of best fit

Interpolation is using the line of best fit to estimate values inside our data point.

e.g. 40 hours revising predicts a percentage of 45



Extrapolation is where we use our line of best fit to predict information outside of our data

This is not always useful – in this example you cannot score more that 100%. So revising for longer can not be estimated

This point is an "**outlier**"
It is an outlier because it doesn't fit this model and stands apart from the data

Averages from lists

The Mean

A measure of average to find the central tendency… a typical value that represents the data

24, 8, 4, 11, 8,

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

55 ÷ 5

Mean = 11

The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8,

This can still be easier if it the data is ordered first

Mode = 8

The Median

The value in the center (in the middle) of the data

24, 8, 4, 11, 8,

Put the data in order

4, 8, 8, 11, 24

Find the value in the middle

4, 8, (8), 11, 24

Median = 8

NOTE: If there is no single middle value find the mean of the two numbers left

For Grouped Data

The modal group – which group has the highest frequency

Averages from a table

Non-grouped data

Number of Siblings	0	1	2
Frequency	6	8	6
Subtotal	0	8	12

Overall Frequency: 20

Total number of siblings: 20

The data in a list: 0,0,0,0,0,0,1,1,1,1,1,1,1,1,2,2,2,2,2,2

Mean: $\frac{\text{total number of siblings}}{\text{Total frequency}} = 1$

Grouped data

x Weight(g)	Frequency	Mid Point	MP x Freq
$40 < x \leq 50$	1	45	45
$50 < x \leq 60$	3	65	195
$60 < x \leq 70$	5	65	325

Overall Frequency: 9

Overall Total: 565

Mean: 62.8g

The data in a list: 45, 55, 55, 55, 65, 65, 65, 65, 65

YEAR 10F — SPRING

A18 - NON-LINEAR GRAPHS

Sparx Maths

Quadratic graphs – U989, U667
Intercepts and roots of quadratic graphs (E) – U601
Cubic graphs – U980
Approximate solutions to equations using graphs (E) – U168

What do I need to be able to do?

Step 1 Quadratic graphs
Step 2 Intercepts and roots of quadratic graphs (E)
Step 3 Cubic graphs
Step 4 Approximate solutions to equations using graphs (E)

Keywords

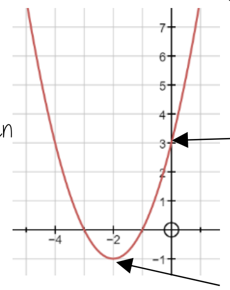
Quadratic: a curved graph with the highest power being 2. Square power.
Inequality: makes a non equal comparison between two numbers
Reciprocal: a reciprocal is 1 divided by the number
Cubic: a curved graph with the highest power being 3. Cubic power.
Origin: the coordinate (0, 0)
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Quadratic graphs are always symmetrical with the turning point in the middle

Substitute the x values into the equation of your line to find the y coordinates

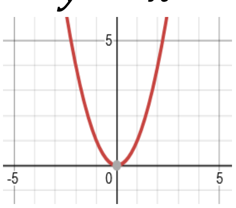
x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

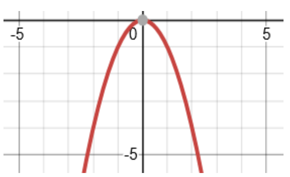
Plot all of the coordinate pairs and join the points with a curve (freehand)

A quadratic graph will always be in the shape of a parabola.

$y = x^2$



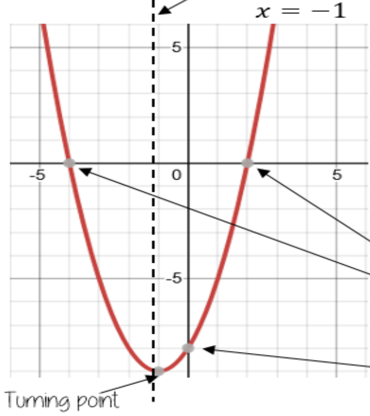
$y = -x^2$



The roots of a quadratic graph are where the graph crosses the x axis. The roots are the solutions to the equation.

Interpreting graphs

Line of symmetry $x = -1$



Examples
$y = x^2 + 2x - 8$

A quadratic equation can be solved from its graph.
The roots of the graph tell us the possible solutions for the equation.
There can be 1 root, 2 roots or no roots for a quadratic equation.
This is dependant on how many times the graph crosses the x axis.

Roots $x = -4$
$x = 2$

y intercept = -8

Turning point $(-1, -9)$

Interpret other graphs

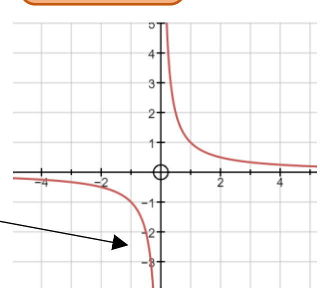
Cubic Graphs

$y = x^3 + 2x^2 - 2x + 1$

If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

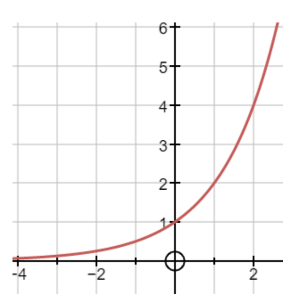


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x



Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 5 (summer 1):

G14 - ANGLES

S6 - GRAPHS AND DIAGRAMS

G15 - VECTORS

YEAR 10F — SUMMER

G14 - ANGLES

Sparx Maths

Angles around a point, on a straight line and vertically opposite – U390, U655, U730
Angles in a triangle – U628 Angles in a quadrilateral – U732
Exterior angles of a polygon – U427 Interior angles of a polygon – U427
Solve problems with angles in polygons (E) – U427, U887
Alternate and corresponding angles – U826
Alternate, corresponding and co-interior angles – U826 Prove geometric facts (E) – U471

What do I need to be able to do?

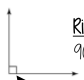
Step 1 Angles around a point, on a straight line and vertically opposite
Step 2 Angles in a triangle
Step 3 Angles in a quadrilateral
Step 4 Exterior angles of a polygon
Step 5 Interior angles of a polygon
Step 6 Solve problems with angles in polygons (E)
Step 7 Alternate and corresponding angles
Step 8 Alternate, corresponding and co-interior angles
Step 9 Prove geometric facts (E)

Keywords

Parallel: Straight lines that never meet
Angle: The figure formed by two straight lines meeting (measured in degrees)
Transversal: A line that cuts across two or more other (normally parallel) lines
Isosceles: Two equal size lines and equal size angles (in a triangle or trapezium)
Polygon: A 2D shape made with straight lines
Sum: Addition (total of all the interior angles added together)
Regular polygon: All the sides have equal length; all the interior angles have equal size.

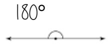
Basic angle rules and notation


Acute Angles
0°< angle <90°

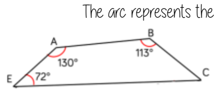

Right Angles
90°
Right angle notation


Obtuse
90°< angle <180°


Reflex
180°< angle <360°


Straight Line
180°

The letter in the middle is the angle
The arc represents the part of the angle



Angle Notation: three letters ABC
This is the angle at B = 113 °
Line Notation: two letters EC
The line that joins E to C

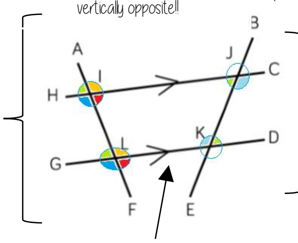

Vertically opposite angles
Equal
Angles around a point
360 °

Parallel lines

Still remember to look for angles on straight lines, around a point and vertically opposite!

Lines AF and BE are **transversals** (lines that bisect the parallel lines)

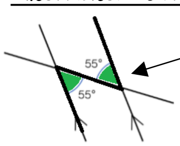
Corresponding angles often identified by their **"F shape"** in position



Alternate angles often identified by their **"Z shape"** in position

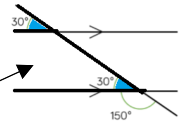
This notation identifies parallel lines

Alternate/ Corresponding angles

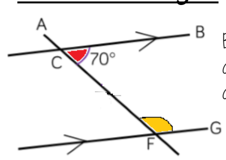


Because alternate angles are equal the highlighted angles are the same size

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

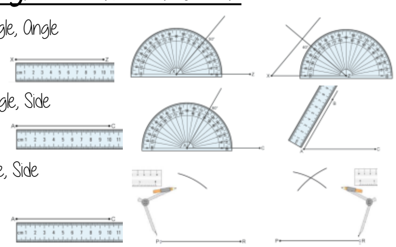


Because co-interior angles have a sum of 180° the highlighted angle is 110°

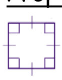
As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

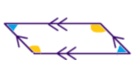
Triangles & Quadrilaterals

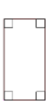
Side, Angle, Angle
Side, Angle, Side
Side, Side, Side

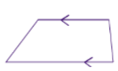


Properties of Quadrilaterals

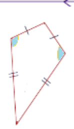

Square
All sides equal size
All angles 90°
Opposite sides are parallel


Parallelogram
Opposite sides are parallel
Opposite angles are equal
Co-interior angles


Rectangle
All angles 90°
Opposite sides are parallel

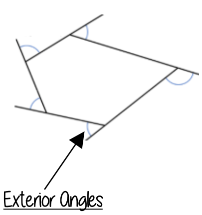

Trapezium
One pair of parallel lines


Rhombus
All sides equal size
Opposite angles are equal


Kite
No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

Sum of exterior angles

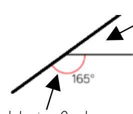
Exterior angles all add up to 360°



Exterior Angles
Are the angle formed from the straight-line extension at the side of the shape

Using exterior angles

Exterior Angle



Interior Angle

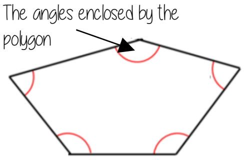
Interior angle + Exterior angle = straight line = 180°
Exterior angle = 180 – 165 = 15°

Number of sides = 360° ÷ exterior angle
Number of sides = 360 ÷ 15 = 24 sides

Sum of interior angles

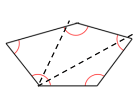
(number of sides – 2) x 180

Interior Angles
The angles enclosed by the polygon



This is an **irregular** polygon – the sides and angles are different sizes

Sum of the interior angles = (5 – 2) x 180

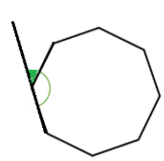


This shape can be made from three triangles
Each triangle has 180°

Sum of the interior angles = 3 x 180
= 540°

Remember this is **all** of the interior angles added together

Missing angles in regular polygons



Exterior angle = 360 ÷ 8 = 45°

Interior angle = $\frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8} = 135°$

Exterior angles in regular polygons = 360° ÷ number of sides

Interior angles in regular polygons = $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

YEAR 10F – SUMMER

S6 - GRAPHS AND DIAGRAMS

Sparx Maths

Pictograms – M644
Line and bar charts – M140 (line), M460 (bar), M738 (interpreting bar charts)
Dual and composite bar charts – U557
Draw pie charts – U508 Interpret pie charts – U172
Time-series graphs – U590 Frequency polygons – U840
Stem-and-leaf diagrams – U200 (drawing), U909 (interpreting)

What do I need to be able to do?

- **Step 1** Pictograms
- **Step 2** Line and bar charts
- **Step 3** Dual and composite bar charts
- **Step 4** Draw pie charts
- **Step 5** Interpret pie charts
- **Step 6** Time-series graphs
- **Step 7** Frequency polygons
- **Step 8** Stem-and-leaf diagrams

Keywords

- **Pictogram** – A chart using pictures or symbols to represent data, with each symbol standing for a set number.
- **Bar Chart** – A graph with bars to show data values; longer bars mean higher values.
- **Line Graph** – A graph showing data points connected by lines to display changes over time.
- **Pie Chart** – A circular chart divided into sectors, each showing a part of the whole in percentages.
- **Axis** – The horizontal or vertical lines on a graph used to plot data.
- **Frequency** – How often a value or category appears in a data set.
- **Dual Bar Chart** – A chart showing two sets of data side-by-side for easy comparison.
- **Time-Series Graph** – A graph that shows how data changes over regular time intervals.
- **Frequency Polygon** – A line graph that joins midpoints of class intervals to show data distribution.
- **Stem-and-Leaf Diagram** – A way to organize numbers by splitting them into "stems" and "leaves".
- **Sector** – A slice of a pie chart representing a part of the total.

Stem and leaf

A way to represent data and use to find averages

This stem and leaf diagram shows the age of people in a line at the supermarket.

0	7 9
1	4 5 6 8 8
2	1 3
3	0

Key: 1|4

Stem and leaf diagrams:
Must include a key to explain what it represents
The information in the diagram should be ordered

Back to back stem and leaf diagrams

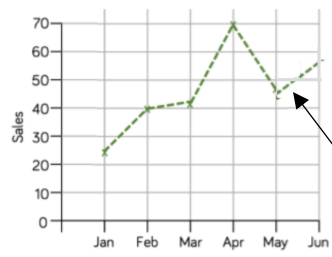
Girls		Boys
5	14	
7, 5, 5, 5, 4	15	3, 8, 9
8, 4, 2, 1, 0	16	2, 5, 7, 7, 7, 8, 8, 9
9, 8, 7, 6, 6, 4, 2, 1, 1, 0, 0	17	0, 2, 3, 6, 6, 7, 7
	18	0, 1, 4, 5

15 | 3
Means 153 cm tall

Back to back stem and leaf diagrams
Allow comparisons of similar groups
Allow representations of two sets of data

Time-Series

This time-series graph shows the total number of car sales in £1000 over time



Look for general trends in the data. Some data shows a clear increase or a clear decrease over time.

Readings in-between points are estimates (on the dotted lines). You can use them to make assumptions.

Draw and interpret Pie Charts

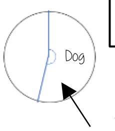
Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey (Total frequency)

$\frac{32}{60}$ "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192°$



Use a protractor to draw
This is 192°

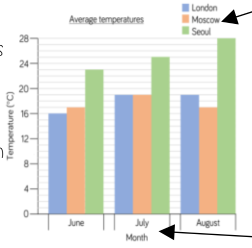
Multiple method
As 60 goes into 360 – 6 times.
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Comparing Pie Charts:
You NEED the overall frequency to make any comparisons

Multiple Bar chart

Compares multiple groups of data

- Clearly labelled axes
- Scale for axes
- Comparable data bars drawn next to each other



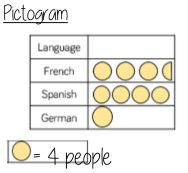
Key/ Colour code for separate groups of information

Gap between different categories of data

Pictograms, bar and line charts

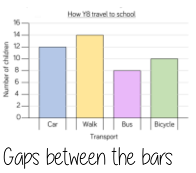
Represents quantitative data

Pictogram



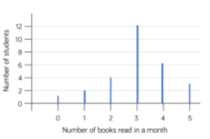
- Need to remember a key
- Visually able to identify mode

Bar Chart



- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

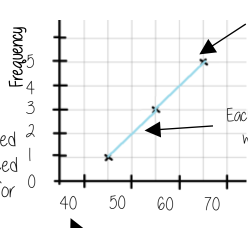
Line Chart



- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

Frequency polygon

x Weight(g)	Frequency
$40 < x \leq 50$	1
$50 < x \leq 60$	3
$60 < x \leq 70$	5



Each point is plotted at them mid point for the group it represents

Each point is connected with a straight line

The data about weight starts at 40. So the axis can start at 40

We do not know from grouped data where each value is placed so have to use an estimate for calculations

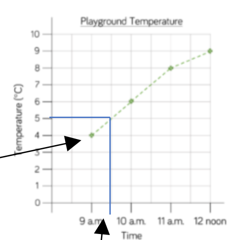
MID POINTS
Mid-points are used as estimated values for grouped data. The middle of each group

Mid-point
$\frac{\text{Start point + End point}}{2}$

Draw and interpret line graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points.

Line graphs do not need to start from 0



More than one piece of data can be plotted on the same graph to compare data

It is possible to make estimates from the line e.g. temperature at 9.30am is 5°C

YEAR 10F — SUMMER

G15 - VECTORS

Sparx Maths

Understand and represent vectors – U632
Vector notation – U632 Translate by a vector – U196
Vectors multiplied by a scalar – U564
Add vectors – U903 Add and subtract vectors – U903
Solve problems with vectors (E) – U781

What do I need to be able to do?

Step 1 Understand and represent vectors
Step 2 Vector notation
Step 3 Translate by a vector
Step 4 Vectors multiplied by a scalar
Step 5 Add vectors
Step 6 Add and subtract vectors
Step 7 Solve problems with vectors (E)

Keywords

Direction: the line our course something is going
Magnitude: the magnitude of a vector is its length
Scalar: a single number used to represent the multiplier when working with vectors
Column vector: a matrix of one column describing the movement from a point
Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet

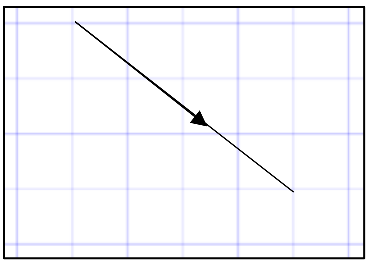
Understand and represent vectors

Vectors show both direction and magnitude

Column vectors have been seen in translations to describe the movement of one image onto another

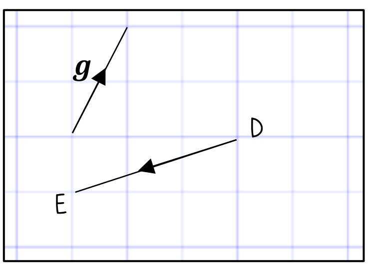
Movement along the x-axis →
Movement along the y-axis ↗

$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ →



The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

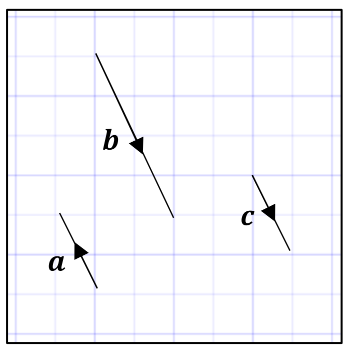
$\overrightarrow{DE} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$. The two lines are parallel

$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

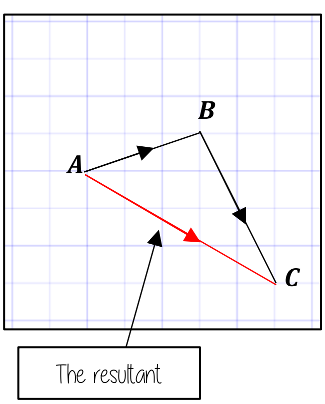
The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{a} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

Addition of vectors

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$



$\overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

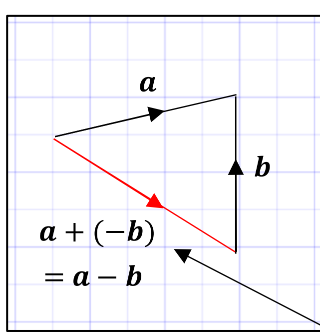
$= \begin{pmatrix} 3+2 \\ 1+-4 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Look how this addition compares to the vector $\overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

Addition and subtraction of vectors



$\boldsymbol{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 5 + -0 \\ 1 + -4 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1

Mathematics Department Vision:

Mathematics provides students with powerful ways to describe, analyse, change and improve the world. The mathematics department at Crawshaw Academy aims to spark a passion in mathematics for all students, no matter what their starting point is, through the beauty of discovering patterns, making connections and looking for the 'why' behind mathematical formulae.

We want our students to:

EXCELLENCE:

•Strive to improve and progress each lesson, allowing themselves to achieve their personal best in mathematics.
•Develop the skills to understand science, technology and engineering as well as everyday tasks essential for keeping safe and healthy and maintaining their own economic well-being.

PURPOSE:

•Tackle rich and diverse problems fluently and make reasoned decisions based on their deep understanding.
•Share our passion for mathematics and have the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
•Apply reason to all that they do, determined to achieve their goals.

AMBITION:

•Strive to develop a curiosity for mathematics through our passion for the subject by having access to mathematics that is both challenging and relevant to everyday life, with an emphasis on problem solving.
•Become fully participating citizens in an ever-changing society who are able to think mathematically, reason and solve problems, and assess risks in a range of contexts.
•Access high quality teaching and learning, so they are encouraged to develop into thinking individuals who are mathematically literate and can achieve their potential.
•Have the desire and enthusiasm to aim higher, with motivation to succeed in our plans for the future.

Year 10 HALF TERM 6 (Summer 2):

N20 - FACTORS AND POWERS

G16 - PYTHAGORAS' THEOREM AND TRIGONOMETRY

A19 - SIMULTANEOUS EQUATIONS

YEAR 10F – SUMMER

N20 - FACTORS AND POWERS

Sparx Maths
Factors, multiples and primes – U211, U236, U751
Prime factorisation – U739
HCF and LCM – U250
Square and cube numbers – U851 (roots and powers), U299 (estimating roots and powers)
Powers and roots – U851

What do I need to be able to do?

Step 1 Factors, multiples and primes
Step 2 Prime factorisation
Step 3 HCF and LCM
Step 4 Square and cube numbers
Step 5 Powers and roots

Keywords

Factor: numbers we multiply together to make another number
Multiple: the result of multiplying a number by an integer.
HCF: highest common factor. The biggest factor that numbers share.
LCM: lowest common multiple. The first multiple numbers share.
Commutative: an operation is commutative if changing the order does not change the result.
Base: The number that gets multiplied by a power
Power: The exponent – or the number that tells you how many times to use the number in multiplication
Exponent: The power – or the number that tells you how many times to use the number in multiplication
Indices: The power or the exponent.
Negative: A value below zero. **Coefficient**: The number used to multiply a variable

Multiples

The "times table" of a given number

All the numbers in this lists below are multiples of 3.

3, 6, 9, 12, 15…

This list continues and doesn't end

$3x, 6x, 9x$ …

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

4.5 is not a multiple of 3 because it is 3 x 1.5 ← Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10: 1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

x	x	x	x	x	x

$6x \times 1$ OR $6 \times x$

Factors of $6x$: 6, x, 1, $6x$, $2x$, 3, $3x$, 2

$2x \times 3$

$3x \times 2$

Prime numbers

- Integer
- Only has 2 factors
- and itself

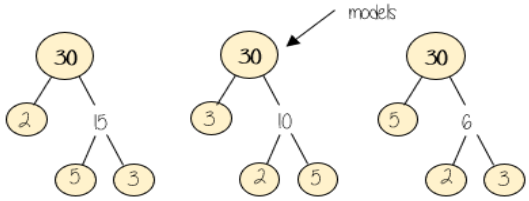
2 — The first prime number. The only even prime number

Learn or how-to quick recall…

2, 3, 5, 7, 11, 13, 17, 19, 23, 29…

Product of prime factors

Multiplication part-whole models



All three prime factor trees represent the same decomposition

30 = 2 x 3 x 5 ← Multiplication of prime factors

Using prime factors for predictions

eg **60** 30 x 2 2 x 3 x 5 **x 2**
150 30 x 5 2 x 3 x 5 **x 5**

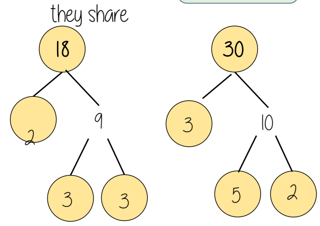
Finding the HCF and LCM

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18
30: 1, 2, 3, 5, 6, 10, 15, 30

6 is the biggest factor they share — HCF = 6

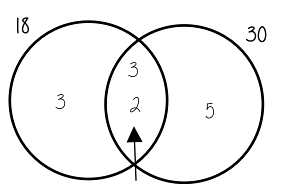


LCM – Lowest common multiple

LCM of 18 and 30

18: 18, 36, 54, 72, 90
30: 30, 60, 90

The first time their multiples match — LCM = 90

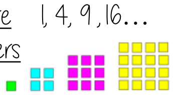


LCM = 90

HCF = 6

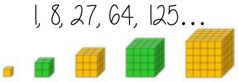
Square and cube numbers

Square numbers 1, 4, 9, 16…



Cube numbers

1, 8, 27, 64, 125…



Addition/ Subtraction Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Zero and negative indices

$x^0 = 1$

Any number divided by itself = 1

$$\frac{a^6}{a^6} = a^6 \div a^6$$
$$= a^{6-6} = a^0 = 1$$

Negative indices do not indicate negative solutions

$2^2 = 4$
$2^1 = 2$
$2^0 = 1$

Looking at the sequence can help to understand negative powers

Powers of powers

$$(x^a)^b = x^{ab}$$

$$(2^3)^4 = 2^3 \times 2^3 \times 2^3 \times 2^3$$

The same base and power is repeated Use the addition law for indices

$$(2^3)^4 = 2^{12}$$ ← NOTICE the difference

$$(2x^3)^4 = 2x^3 \times 2x^3 \times 2x^3 \times 2x^3$$

The addition law applies ONLY to the powers. The integers still need to be multiplied

$$(2x^3)^4 = 16x^{12}$$

Higher powers and roots

x^n

n – power (number of times multiplied by itself)

x – the base number.

$\sqrt[n]{x}$ — Finding the nth root of any value

Other mental strategies for square roots

$$\sqrt{810000} = \sqrt{81} \times \sqrt{10000}$$
$$= 9 \times 100$$
$$= 900$$

YEAR 10F — SUMMER

G16 - PYTHAGORAS' THEOREM AND TRIGONOMETRY

CRAWSHAW MATHEMATICS

Sparx Maths

Pythagoras' theorem (find the hypotenuse) – U385 Pythagoras' theorem (find any side) – U828
Identify hypotenuse, opposite and adjacent sides – U283 Ratios in right-angled triangles – U605
Use the tangent ratio to find an unknown side length – U283
Use the sine and cosine ratio to find an unknown side length – U283
Use trigonometric ratios to find an unknown side length – U283
Use trigonometric ratios to find an unknown angle – U545 Exact trigonometric values (E) – U627

What do I need to be able to do?

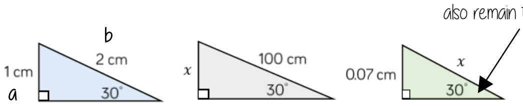
Step 1 Pythagoras' theorem (find the hypotenuse)
Step 2 Pythagoras' theorem (find any side)
Step 3 Identify hypotenuse, opposite and adjacent sides
Step 4 Ratios in right-angled triangles
Step 5 Use the tangent ratio to find an unknown side length
Step 6 Use the sine and cosine ratio to find an unknown side length
Step 7 Use trigonometric ratios to find an unknown side length
Step 8 Use trigonometric ratios to find an unknown angle
Step 9 Exact trigonometric values (E)

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Constant: a value that remains the same
Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.
Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.
Inverse: function that has the opposite effect.
Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

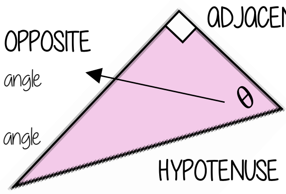
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

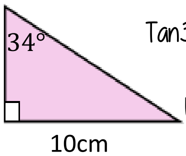
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



$\text{Tan}34 = \frac{10}{x}$

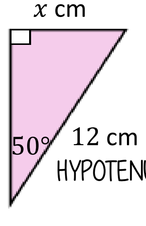
Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

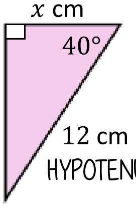
Sin and Cos ratio: side lengths

$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$



NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio

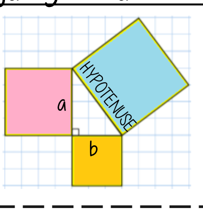
$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$



Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem



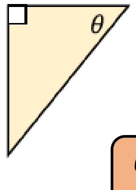
$\text{Hypotenuse}^2 = a^2 + b^2$

This is commutative – the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\text{Tan}\theta = \frac{3}{4}$

$\theta = \text{Tan}^{-1}\frac{3}{4}$

$\theta = 36.9°$

$\theta = \text{Tan}^{-1}\frac{\text{opposite side}}{\text{adjacent side}}$

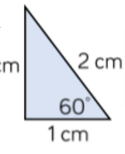
$\theta = \text{Sin}^{-1}\frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements

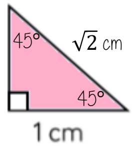
This side could be calculated using Pythagoras



$\text{Tan}30 = \frac{1}{\sqrt{3}}$
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$
$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$
$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

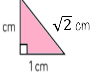
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

Tan0 = 0

~~Tan90~~

This value cannot be defined – it is impossible as you cannot have two 90° angles in a triangle



Sin0 = 0

Sin90 = 1

Cos0 = 1

Cos90 = 0

YEAR 10F – SUMMER

A19 - SIMULTANEOUS EQUATIONS

Sparx Maths

Solve simultaneous equations using graphs – U836
Solve simultaneous equations (no adjustments) – U760
Solve simultaneous equations (adjust one) – U760
Solve simultaneous equations (adjust both) (E) – U760
Solve simultaneous equations by substitution (E) – U757

What do I need to be able to do?

Step 1 Use one value to find another
Step 2 Introduction to simultaneous equations
Step 3 Solve simultaneous equations using graphs
Step 4 Solve simultaneous equations (no adjustments)
Step 5 Manipulating equations
Step 6 Solve simultaneous equations (adjust one)
Step 7 Solve simultaneous equations (adjust both) (E)
Step 8 Solve simultaneous equations by substitution (E)

Keywords

Solution: a value we can put in place of a variable that makes the equation true
Variable: a symbol for a number we don't know yet
Equation: an equation says that two things are equal – it will have an equals sign =
Substitute: replace a variable with a numerical value
LCM: lowest common multiple (the first time the times table of two or more numbers match)
Eliminate: to remove
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
Coordinate: a set of values that show an exact position
Intersection: the point two lines cross or meet

Is (x, y) a solution?

x and y represent values that can be substituted into an equation

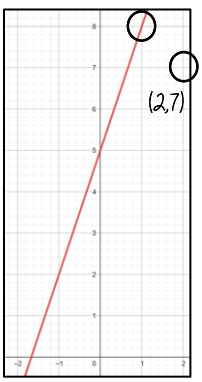
Does the coordinate (1,8) lie on the line y=3x+5?

This coordinate represents x=1 and y=8

$y = 3x + 5$

$8 = 3(1) + 5$

As the substitution makes the equation correct the coordinate (1,8) IS on the line y=3x+5



(2,7)

Is (2,7) on the same line?

$7 \neq 3(2) + 5$

No 7 does NOT equal 6+5

Substituting known variables

A line has the equation 3x + y = 14

Stephanie knows the point x = 4 lies on that line. Find the value for y

$x = 4$

$3x + y = 14$ [x | x | x | y = 14]

$3(4) + y = 14$ [4 | 4 | 4 | y = 14]

Two different variables, two solutions

$12 + y = 14$
$-12 \quad -12$
$y = 2$

Substituting in an expression

x = 2y

x + y = 30

Pair of simultaneous equations (two representations)

Substitute 2y in place of the x variable as they represent the same value

$x = 2y$ $\quad x + y = 30$

$3y = 30$

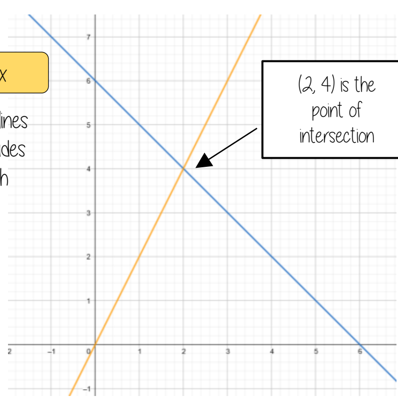
$3y = 30$
$\div 3 \quad \div 3$
$y = 10$

$x = 2y$
$x = 20$

Solve graphically

x + y = 6 $\quad$ y = 2x

Linear equations are straight lines
The point of intersection provides the x and y solution for both equations



(2, 4) is the point of intersection

The solution that satisfies both equations is

$x = 2$ and $y = 4$

Solve by subtraction

$3x + 2y = 18$
$-\quad x + 2y = 10$

$2x = 8$
$\div 2 \quad \div 2$
$x = 4$

$x + 2y = 10$
$(4) + 2y = 10$
$-4 \quad -4$
$2y = 6$
$\div 2 \quad \div 2$
$y = 3$

$x = 4$
$y = 3$

= 18
= 10
= 18
= 10
= 8
= 4
= 3

Solve by addition

Addition makes zero pairs

$3x + 2y = 16$
$+\ 6x - 2y = 2$

$9x = 18$
$\div 9 \quad \div 9$
$x = 2$

$3x + 2y = 16$
$3(2) + 2(y) = 16$
$6 + 2y = 16$
$-6 \quad -6$
$2y = 10$
$y = 5$

= 16
= 2
= 18
= 2
= 5

Solve by adjusting one

$h + j = 12$
$2h + 2j = 29$

No equivalent values

12
29

$2h + 2j = 24$
$2h + 2j = 29$

24
29

By proportionally adjusting one of the equations – now solve the simultaneous equations choosing an addition or subtraction method

Solve by adjusting both

$2x + 3y = 39$
$5x - 2y = -7$

= 39
= −7

Use LCM to make equivalent x OR y values. Because of the negative values using zero pairs and y values is chosen choice

$4x + 6y = 78$
$15x - 6y = -21$

= 78
= −21

Now solve by addition

Addition makes zero pairs